1 The cubic equation $x^3 + bx^2 + cx + d = 0$, where b, c and d are constants, has roots α, β, γ. It is given that $\alpha\beta\gamma = -1$.

(a) State the value of d. [1]

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(b) Find a cubic equation, with coefficients in terms of b and c, whose roots are $\alpha+1$, $\beta+1$, $\gamma+1$. [3]

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(c) Given also that $\gamma + 1 = -\alpha - 1$, deduce that $(c - 2b + 3)(b - 3) = b - c$. [4]

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2 Prove by mathematical induction that $7^{2n} - 1$ is divisible by 12 for every positive integer n. [5]

3 **(a)** By simplifying $\left(x^n-\sqrt{x^{2n}+1}\right)\left(x^n+\sqrt{x^{2n}+1}\right)$, show that $\dfrac{1}{x^n-\sqrt{x^{2n}+1}}=-x^n-\sqrt{x^{2n}+1}$. [1]

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Let $u_n=x^{n+1}+\sqrt{x^{2n+2}+1}+\dfrac{1}{x^n-\sqrt{x^{2n}+1}}$.

(b) Use the method of differences to find $\sum_{n=1}^{N}u_n$ in terms of N and x. [3]

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(c) Deduce the set of values of x for which the infinite series

$$u_1+u_2+u_3+\dots$$

is convergent and give the sum to infinity when this exists. [3]

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4 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}.$$

(a) Give full details of the geometrical transformation in the x-y plane represented by **A**. [1]

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(b) Give full details of the geometrical transformation in the x-y plane represented by **B**. [2]

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The triangle DEF in the x-y plane is transformed by **AB** onto triangle PQR.

(c) Show that the triangles DEF and PQR have the same area. [3]

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(d) Find the matrix which transforms triangle *PQR* onto triangle *DEF*. [2]

(e) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **AB**. [5]

5 The curve C has polar equation $r = \ln(1 + \pi - \theta)$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C and state the polar coordinates of the point of C furthest from the pole. [3]

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(b) Using the substitution $u = 1 + \pi - \theta$, or otherwise, show that the area of the region enclosed by C and the initial line is

$$\tfrac{1}{2}(1+\pi)\ln(1+\pi)\big(\ln(1+\pi)-2\big)+\pi.$$ [6]

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(c) Show that, at the point of C furthest from the initial line,

$$(1+\pi-\theta)\ln(1+\pi-\theta)-\tan\theta=0$$

and verify that this equation has a root between 1.2 and 1.3. [5]

6 Let a be a positive constant.

(a) The curve C_1 has equation $y = \frac{x-a}{x-2a}$. [2]

Sketch C_1.

The curve C_2 has equation $y = \left(\frac{x-a}{x-2a}\right)^2$. The curve C_3 has equation $y = \left|\frac{x-a}{x-2a}\right|$.

(b) **(i)** Find the coordinates of any stationary points of C_2. [3]

(ii) Find also the coordinates of any points of intersection of C_2 and C_3. [3]

(c) Sketch C_2 and C_3 on a single diagram, clearly identifying each curve. Hence find the set of values of x for which $\left(\frac{x-a}{x-2a}\right)^2 \leqslant \left|\frac{x-a}{x-2a}\right|$. [5]

7 The points A, B, C have position vectors

$$-2\mathbf{i}+2\mathbf{j}-\mathbf{k}, \quad -2\mathbf{i}+\mathbf{j}+2\mathbf{k}, \quad -2\mathbf{j}+\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

(b) Find the acute angle between the planes OBC and ABC. [4]

The point D has position vector $t\mathbf{i}-\mathbf{j}$.

(c) Given that the shortest distance between the lines AB and CD is $\sqrt{10}$, find the value of t. [6]