

1 The cubic equation $x^3 + bx^2 + cx + d = 0$, where b , c and d are constants, has roots α , β , γ . It is given that $\alpha\beta\gamma = -1$.

(a) State the value of d . [1]

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(b) Find a cubic equation, with coefficients in terms of b and c , whose roots are $\alpha + 1$, $\beta + 1$, $\gamma + 1$. [3]

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(c) Given also that $\gamma + 1 = -\alpha - 1$, deduce that $(c - 2b + 3)(b - 3) = b - c$. [4]

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- 3 (a) By simplifying $(x^n - \sqrt{x^{2n} + 1})(x^n + \sqrt{x^{2n} + 1})$, show that $\frac{1}{x^n - \sqrt{x^{2n} + 1}} = -x^n - \sqrt{x^{2n} + 1}$. [1]

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Let $u_n = x^{n+1} + \sqrt{x^{2n+2} + 1} + \frac{1}{x^n - \sqrt{x^{2n} + 1}}$.

- (b) Use the method of differences to find $\sum_{n=1}^N u_n$ in terms of N and x . [3]

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- (c) Deduce the set of values of x for which the infinite series

$$u_1 + u_2 + u_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

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4 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}.$$

(a) Give full details of the geometrical transformation in the *x-y* plane represented by **A**. [1]

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(b) Give full details of the geometrical transformation in the *x-y* plane represented by **B**. [2]

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The triangle *DEF* in the *x-y* plane is transformed by **AB** onto triangle *PQR*.

(c) Show that the triangles *DEF* and *PQR* have the same area. [3]

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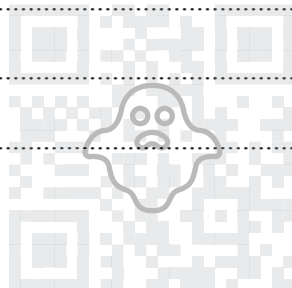
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5 The curve C has polar equation $r = \ln(1 + \pi - \theta)$, for $0 \leq \theta \leq \pi$.

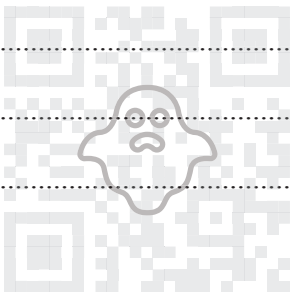
(a) Sketch C and state the polar coordinates of the point of C furthest from the pole. [3]

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(b) Using the substitution $u = 1 + \pi - \theta$, or otherwise, show that the area of the region enclosed by C and the initial line is

$$\frac{1}{2}(1 + \pi)\ln(1 + \pi)(\ln(1 + \pi) - 2) + \pi. \quad [6]$$

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(c) Show that, at the point of C furthest from the initial line,

$$(1 + \pi - \theta)\ln(1 + \pi - \theta) - \tan \theta = 0$$

and verify that this equation has a root between 1.2 and 1.3. [5]

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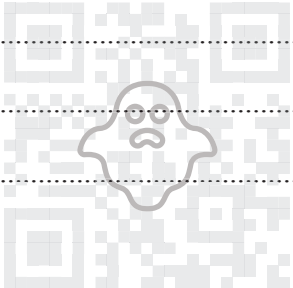
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6 Let a be a positive constant.

(a) The curve C_1 has equation $y = \frac{x-a}{x-2a}$. [2]

Sketch C_1 .

The curve C_2 has equation $y = \left(\frac{x-a}{x-2a}\right)^2$. The curve C_3 has equation $y = \left|\frac{x-a}{x-2a}\right|$.

(b) (i) Find the coordinates of any stationary points of C_2 . [3]

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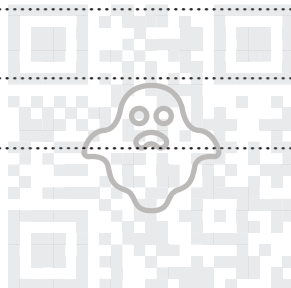
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(ii) Find also the coordinates of any points of intersection of C_2 and C_3 . [3]

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(c) Sketch C_2 and C_3 on a single diagram, clearly identifying each curve. Hence find the set of values of x for which $\left(\frac{x-a}{x-2a}\right)^2 \leq \left|\frac{x-a}{x-2a}\right|$. [5]



7 The points A, B, C have position vectors

$$-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}, \quad -2\mathbf{i} + \mathbf{j} + 2\mathbf{k}, \quad -2\mathbf{j} + \mathbf{k},$$

respectively, relative to the origin O .

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

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(b) Find the acute angle between the planes OBC and ABC . [4]

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