

1 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}$, where *a* and *b* are positive constants.

(a) The matrix **M** represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

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The unit square in the *x-y* plane is transformed by **M** onto parallelogram *OPQR*.

(b) Find, in terms of *a* and *b*, the matrix which transforms parallelogram *OPQR* onto the unit square. [2]

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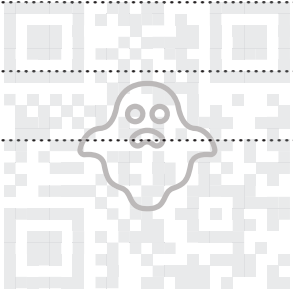
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- (b) Use the method of differences to find $\sum_{r=1}^n \frac{1}{(7r+1)(7r+8)}$ in terms of n . [4]

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- (c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(7r+1)(7r+8)}$. [1]

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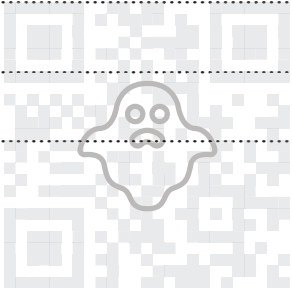
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3 The cubic equation $x^3 + cx + 1 = 0$, where c is a constant, has roots α, β, γ .

(a) Find a cubic equation whose roots are $\alpha^3, \beta^3, \gamma^3$. [3]

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(b) Show that $\alpha^6 + \beta^6 + \gamma^6 = 3 - 2c^3$. [3]

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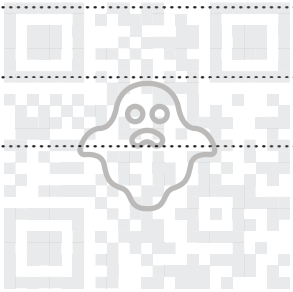
4 The points A, B, C have position vectors

$$-\mathbf{i}+\mathbf{j}+2\mathbf{k}, \quad -2\mathbf{i}-\mathbf{j}, \quad 2\mathbf{i}+2\mathbf{k},$$

respectively, relative to the origin O .

(a) Find the equation of the plane ABC , giving your answer in the form $ax+by+cz=d$. [5]

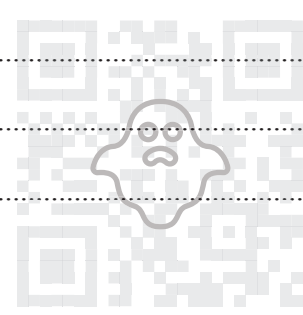
Dotted lines for writing the solution.



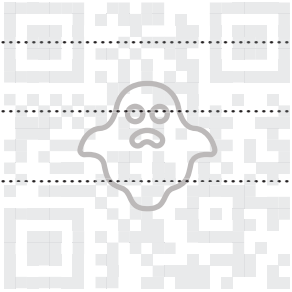
5 Prove by mathematical induction that, for every positive integer n ,

$$\frac{d^{2n-1}}{dx^{2n-1}}(x \sin x) = (-1)^{n-1}(x \cos x + (2n - 1) \sin x). \quad [7]$$

Dotted lines for writing the proof.



Dotted lines for writing.



6 The curve C has equation $y = \frac{x^2 + x - 1}{x - 1}$.

(a) Find the equations of the asymptotes of C . [3]

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(b) Show that there is no point on C for which $1 < y < 5$. [4]

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(c) Find the coordinates of the intersections of C with the axes, and sketch C . [3]

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(d) Sketch the curve with equation $y = \left| \frac{x^2 + x - 1}{x - 1} \right|$. [2]



The curve C has polar equation $r = \sin 4\theta$, for $0 \leq \theta \leq \frac{1}{4}\pi$.

(b) Sketch C and state the equation of the line of symmetry. [3]

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(c) Find the exact value of the area of the region enclosed by C . [4]

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