1 A manager is investigating the times taken by employees to complete a particular task as a result of the introduction of new technology. He claims that the mean time taken to complete the task is reduced by more than 0.4 minutes. He chooses a random sample of 10 employees. The times taken, in minutes, before and after the introduction of the new technology are recorded in the table.

Employee	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Time before new technology	10.2	9.8	12.4	11.6	10.8	11.2	14.6	10.6	12.3	11.0
Time after new technology	9.6	8.5	12.4	10.9	10.2	10.6	12.8	10.8	12.5	10.6

(a) Test at the 10% significance level whether the manager's claim is justified. [7]

(b) State an assumption that is necessary for this test to be valid. [1]

2 The probability generating function, $G_Y(t)$, of the random variable Y is given by

$$G_Y(t) = 0.04 + 0.2t + 0.37t^2 + 0.3t^3 + 0.09t^4.$$

(a) Find Var(Y). [4]

The random variable Y is the sum of two independent observations of the random variable X.

(b) Find the probability generating function of X, giving your answer as a polynomial in t. [3]

3 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} kx(4-x) & 0 \leqslant x < 2, \\ k(6-x) & 2 \leqslant x \leqslant 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{3}{40}$. [1]

(b) Given that $\mathrm{E}(X) = 2.5$, find $\mathrm{Var}(X)$. [3]

(c) Find the median value of X. [4]

4 A scientist is investigating the numbers of a particular type of butterfly in a certain region. He claims that the numbers of these butterflies found per square metre can be modelled by a Poisson distribution with mean 2.5. He takes a random sample of 120 areas, each of one square metre, and counts the number of these butterflies in each of these areas. The following table shows the observed frequencies together with some of the expected frequencies using the scientist's Poisson distribution.

Number per square metre	0	1	2	3	4	5	6	$\geqslant 7$
Observed frequency	12	20	36	32	13	6	1	0
Expected frequency	9.85	24.63	30.78	25.65	p	8.02	3.34	q

(a) Find the values of p and q, correct to 2 decimal places. [2]

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(b) Carry out a goodness of fit test, at the 10% significance level, to test the scientist's claim. [6]

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5 Raman is researching the heights of male giraffes in a particular region. Raman assumes that the heights of male giraffes in this region are normally distributed. He takes a random sample of 8 male giraffes from the region and measures the height, in metres, of each giraffe. These heights are as follows.

5.2 5.8 4.9 6.1 5.5 5.9 5.4 5.6

(a) Find a 90% confidence interval for the population mean height of male giraffes in this region. [5]

Raman claims that the population mean height of male giraffes in the region is less than 5.9 metres.

(b) Test at the 2.5% significance level whether this sample provides sufficient evidence to support Raman's claim. [4]

6 A teacher at a large college gave a mathematical puzzle to all the students. The median time taken by a random sample of 24 students to complete the puzzle was 18.0 minutes. The students were then given practice in solving puzzles. Two weeks later, the students were given another mathematical puzzle of the same type as the first. The times, in minutes, taken by the random sample of 24 students to complete this puzzle are as follows.

18.2	17.5	16.4	15.1	20.5	26.5	19.2	23.2
17.9	18.8	25.8	19.9	17.7	16.2	17.3	16.6
17.1	20.1	20.3	12.6	16.0	21.4	22.7	18.4

The teacher claims that the practice has not made any difference to the average time taken to complete a puzzle of this type.

Carry out a Wilcoxon signed-rank test, at the 10% significance level, to test whether there is sufficient evidence to reject the teacher's claim. [10]