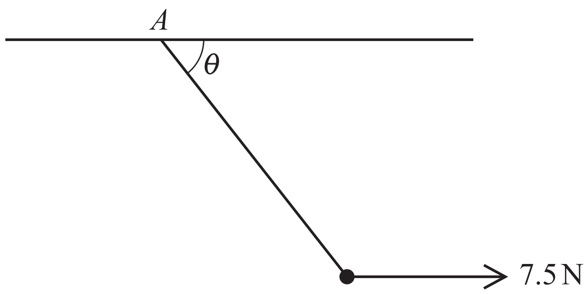
1




A particle of weight 10 N is attached to one end of a light elastic string. The other end of the string is attached to a fixed point $A$ on a horizontal ceiling. A horizontal force of 7.5 N acts on the particle. In the equilibrium position, the string makes an angle $\theta$ with the ceiling (see diagram). The string has natural length 0.8 m and modulus of elasticity 50 N.

**(a)** Find the tension in the string. [2]

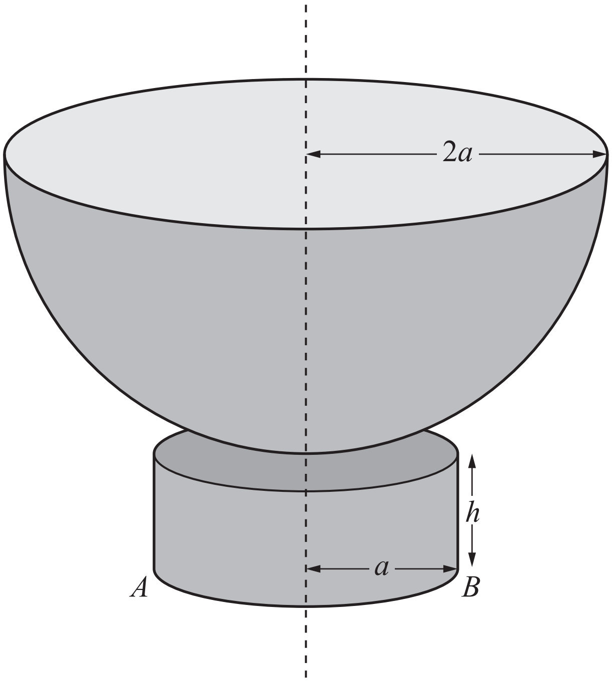
**(b)** Find the vertical distance between the particle and the ceiling. [3]

2 One end of a light inextensible string of length $a$ is attached to a fixed point $O$. A particle of mass $m$ is attached to the other end of the string. The particle is held at the point $A$ with the string taut. The angle between $OA$ and the downward vertical is equal to $\alpha$, where $\cos\alpha = \frac{4}{5}$. The particle is projected from $A$, perpendicular to the string in an upwards direction, with a speed $\sqrt{3ga}$. It then moves along a circular path in a vertical plane. The string first goes slack when it makes an angle $\theta$ with the upward vertical through $O$.

Find the value of $\cos\theta$. [5]

**3** A particle $P$ is moving in a horizontal straight line. Initially $P$ is at the point $O$ on the line and is moving with velocity $25\,\mathrm{m\,s^{-1}}$. At time $t$ s after passing through $O$, the acceleration of $P$ is $\frac{4000}{(5t+4)^3}\,\mathrm{m\,s^{-2}}$ in the direction $PO$. The displacement of $P$ from $O$ at time $t$ is $x$ m.

Find an expression for $x$ in terms of $t$. [5]




An object is composed of a hemispherical shell of radius $2a$ attached to a closed hollow circular cylinder of height $h$ and base radius $a$. The hemispherical shell and the hollow cylinder are made of the same uniform material. The axes of symmetry of the shell and the cylinder coincide. $AB$ is a diameter of the lower end of the cylinder (see diagram).

**(a)** Find, in terms of $a$ and $h$, an expression for the distance of the centre of mass of the object from $AB$. [4]

........................................................................................................................................

........................................................................................................................................

........................................................................................................................................

........................................................................................................................................

........................................................................................................................................

........................................................................................................................................

........................................................................................................................................

........................................................................................................................................

........................................................................................................................................

........................................................................................................................................

........................................................................................................................................

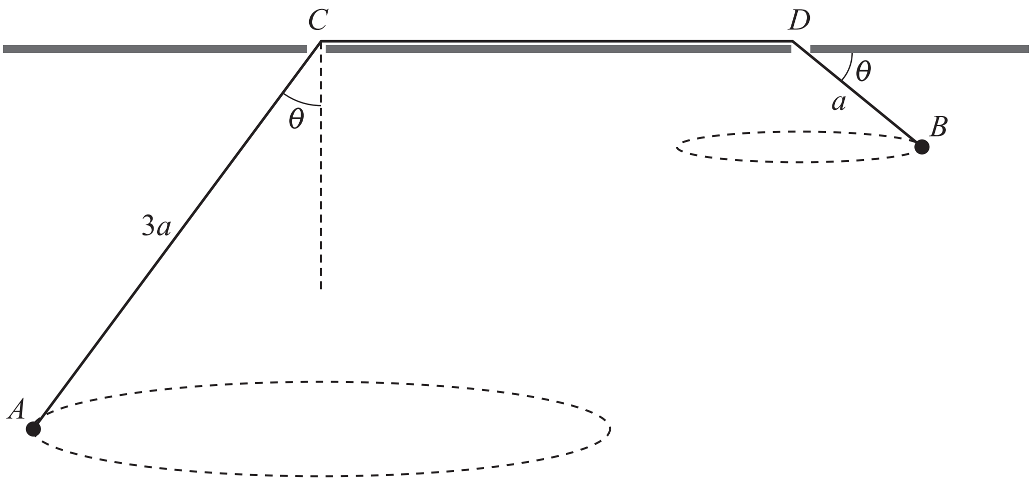
........................................................................................................................................

........................................................................................................................................

The object is placed on a rough plane which is inclined to the horizontal at an angle $\theta$, where $\tan\theta = \frac{2}{3}$. The object is in equilibrium with $AB$ in contact with the plane and lying along a line of greatest slope of the plane.

**(b)** Find the set of possible values of $h$, in terms of $a$. [4]

**5**




A light inextensible string $AB$ passes through two small holes $C$ and $D$ in a smooth horizontal table where $AC = 3a$ and $DB = a$. A particle of mass $m$ is attached at the end $A$ and moves in a horizontal circle with angular velocity $\omega$. A particle of mass $\frac{3}{4}m$ is attached to the end $B$ and moves in a horizontal circle with angular velocity $k\omega$. $AC$ makes an angle $\theta$ with the downward vertical and $DB$ makes an angle $\theta$ with the horizontal (see diagram).

Find the value of $k$. [7]

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

................................................................................................................................................

6 Two uniform smooth spheres $A$ and $B$ of equal radii have masses $m$ and $km$ respectively. The two spheres are on a horizontal surface. Sphere $A$ is travelling with speed $u$ towards sphere $B$ which is at rest. The spheres collide. Immediately before the collision, the direction of motion of $A$ makes an angle $\alpha$ with the line of centres. The coefficient of restitution between the spheres is $\frac{1}{2}$.

**(a)** Show that the speed of $B$ after the collision is $\dfrac{3u\cos\alpha}{2(1+k)}$ and find also an expression for the speed of $A$ along the line of centres after the collision, in terms of $k$, $u$ and $\alpha$. [4]

After the collision, the kinetic energy of $A$ is equal to the kinetic energy of $B$.

**(b)** Given that $\tan\alpha = \frac{2}{3}$, find the possible values of $k$. [5]

**7** Particles $P$ and $Q$ are projected in the same vertical plane from a point $O$ at the top of a cliff. The height of the cliff exceeds 50 m. Both particles move freely under gravity. Particle $P$ is projected with speed $\frac{35}{2}\,\mathrm{m\,s^{-1}}$ at an angle $\alpha$ above the horizontal, where $\tan\alpha = \frac{4}{3}$. Particle $Q$ is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle $\beta$ above the horizontal, where $\tan\beta = \frac{1}{2}$. Particle $Q$ is projected one second after the projection of particle $P$. The particles collide $T$ s after the projection of particle $Q$.

**(a)** Write down expressions, in terms of $T$, for the horizontal displacements of $P$ and $Q$ from $O$ when they collide and hence show that $4uT = 21\sqrt{5}(T+1)$. [4]

**(b)** Find the value of $T$. [4]

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

**(c)** Find the horizontal and vertical displacements of the particles from $O$ when they collide. [3]

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................

..............................................................................................................................................