

- 1 (a) Sketch the curve with equation  $y = \frac{x+1}{x-1}$ . [2]

- (b) Sketch the curve with equation  $y = \frac{|x|+1}{|x|-1}$  and find the set of values of  $x$  for which  $\frac{|x|+1}{|x|-1} < -2$ . [4]







3 A curve  $C$  has equation  $y = \frac{ax^2 + x - 1}{x - 1}$ , where  $a$  is a positive constant.

(a) Find the equations of the asymptotes of  $C$ . [3]

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(b) Show that there is no point on  $C$  for which  $1 < y < 1 + 4a$ . [4]

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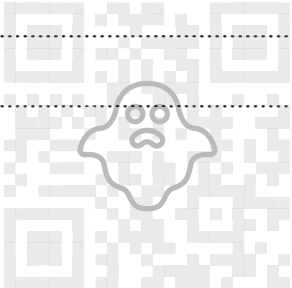
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(c) Sketch *C*. You do not need to find the coordinates of the intersections with the axes. [3]



4 Let  $u_r = e^{rx}(e^{2x} - 2e^x + 1)$ .

(a) Using the method of differences, or otherwise, find  $\sum_{r=1}^n u_r$  in terms of  $n$  and  $x$ . [3]

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(b) Deduce the set of non-zero values of  $x$  for which the infinite series

$$u_1 + u_2 + u_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

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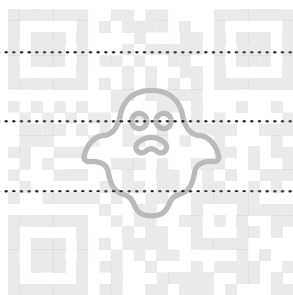
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- (c) Using a standard result from the list of formulae (MF19), find  $\sum_{r=1}^n \ln u_r$  in terms of  $n$  and  $x$ . [3]



5 Let  $A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ , where  $a$  is a positive constant.

- (a) State the type of the geometrical transformation in the  $x$ - $y$  plane represented by  $A$ . [1]

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- (b) Prove by mathematical induction that, for all positive integers  $n$ ,

$$A^n = \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}. \quad [5]$$

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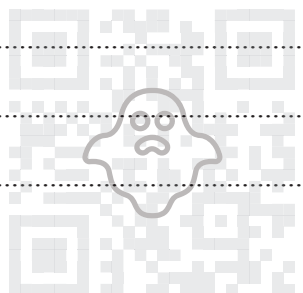
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Let  $\mathbf{B} = \begin{pmatrix} b & b \\ a^{-1} & a^{-1} \end{pmatrix}$ , where  $b$  is a positive constant.

- (c) Find the equations of the invariant lines, through the origin, of the transformation in the  $x$ - $y$  plane represented by  $\mathbf{A}^n \mathbf{B}$ . [6]

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6 The curve  $C$  has Cartesian equation  $x^2 + xy + y^2 = a$ , where  $a$  is a positive constant.

(a) Show that the polar equation of  $C$  is  $r^2 = \frac{2a}{2 + \sin 2\theta}$ . [3]

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(b) Sketch the part of  $C$  for  $0 \leq \theta \leq \frac{1}{4}\pi$ . [2]



The region  $R$  is enclosed by this part of  $C$ , the initial line and the half-line  $\theta = \frac{1}{4}\pi$ .

- (c) It is given that  $\sin 2\theta$  may be expressed as  $\frac{2 \tan \theta}{1 + \tan^2 \theta}$ . Use this result to show that the area of  $R$  is

$$\frac{1}{2}a \int_0^{\frac{1}{4}\pi} \frac{1 + \tan^2 \theta}{1 + \tan \theta + \tan^2 \theta} d\theta$$

and use the substitution  $t = \tan \theta$  to find the exact value of this area.

[8]

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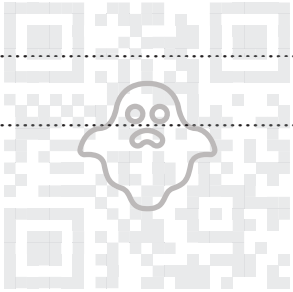
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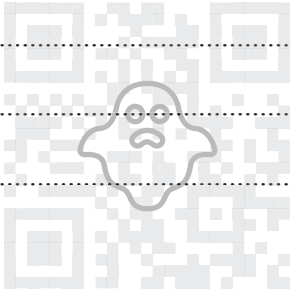
7 The position vectors of the points  $A, B, C, D$  are

$7\mathbf{i} + 4\mathbf{j} - \mathbf{k}, \quad 11\mathbf{i} + 3\mathbf{j}, \quad 2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}, \quad 2\mathbf{i} + 7\mathbf{j} + \lambda\mathbf{k}$

respectively.

(a) Given that the shortest distance between the line  $AB$  and the line  $CD$  is 3, show that  $\lambda^2 - 5\lambda + 4 = 0$ . [7]

Dotted lines for writing the answer.



Let  $\Pi_1$  be the plane  $ABD$  when  $\lambda = 1$ .

Let  $\Pi_2$  be the plane  $ABD$  when  $\lambda = 4$ .

(b) (i) Write down an equation of  $\Pi_1$ , giving your answer in the form  $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$ . [2]

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(ii) Find an equation of  $\Pi_2$ , giving your answer in the form  $ax + by + cz = d$ . [4]

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