1 Farmer A grows apples of a certain variety. Each tree produces 14.8 kg of apples, on average, per year. Farmer B grows apples of the same variety and claims that his apple trees produce a higher mass of apples per year than Farmer A's trees. The masses of apples from Farmer B's trees may be assumed to be normally distributed.

A random sample of 10 trees from Farmer B is chosen. The masses, x kg, of apples produced in a year are summarised as follows.

$$\sum x = 152.0 \qquad \sum x^2 = 2313.0$$

Test, at the 5% significance level, whether Farmer B's claim is justified. [6]

2 A company is developing a new flavour of chocolate by varying the quantities of the ingredients. A random selection of 9 flavours of chocolate are judged by two tasters who each give marks out of 100 to each flavour of chocolate.

Chocolate	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
Taster 1	72	86	75	92	98	79	87	60	62
Taster 2	84	72	74	95	85	87	82	75	68

Carry out a Wilcoxon matched-pairs signed-rank test at the 10% significance level to investigate whether, on average, there is a difference between marks awarded by the two tasters. [7]

3 The heights, x m, of a random sample of 50 adult males from country A were recorded. The heights, y m, of a random sample of 40 adult males from country B were also recorded. The results are summarised as follows.

$$\Sigma x = 89.0 \qquad \Sigma x^2 = 159.4 \qquad \Sigma y = 67.2 \qquad \Sigma y^2 = 113.1$$

Find a 95% confidence interval for the difference between the mean heights of adult males from country A and adult males from country B. [8]

4 X is a discrete random variable which takes the values 0, 2, 4, … . The probability generating function of X is given by

$$G_X(t) = \frac{1}{3-2t^2}.$$

(a) Find $E(X)$ and $Var(X)$. [5]

(b) Find $P(X = 4)$. [3]

5 Chai packs china mugs into cardboard boxes. Chai's manager suspects that breakages occur at random times and that the number of breakages may follow a Poisson distribution. He takes a small sample of observations and finds that the number of breakages in a one-hour period has a mean of 2.4 and a standard deviation of 1.5.

(a) Explain how this information tends to support the manager's suspicion. [2]

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The manager now takes a larger sample and claims that the numbers of breakages in a one-hour period follow a Poisson distribution. The numbers of breakages in a random sample of 180 one-hour periods are summarised in the following table.

Number of breakages	0	1	2	3	4	5	6	7 or more
Frequency	21	33	46	31	23	16	10	0

The mean number of breakages calculated from this sample is 2.5.

(b) Use the data from this larger sample to carry out a goodness of fit test, at the 10% significance level, to test the claim. [8]

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6 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{8} & 0 \leqslant x < 1, \\ \frac{1}{28}(8-x) & 1 \leqslant x \leqslant 8, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

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(b) Find the value of the constant a such that $P(X \leqslant a) = \frac{5}{7}$. [3]

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The random variable Y is given by $Y = \sqrt[3]{X}$.

(c) Find the probability density function of Y. [5]