

1 A random sample of 7 observations of a variable X are as follows.

8.26 7.78 7.92 8.04 8.27 7.95 8.34

The population mean of X is μ .

(a) Test, at the 10% significance level, the null hypothesis $\mu = 8.22$ against the alternative hypothesis $\mu < 8.22$. [6]

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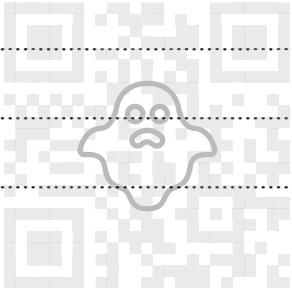
(b) State an assumption necessary for the test in part (a) to be valid. [1]

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3 The continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{81}x^2 & 0 \leq x \leq 9, \\ 1 & x > 9. \end{cases}$$

(a) Find $E(\sqrt{X})$. [3]

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(b) Find $\text{Var}(\sqrt{X})$. [2]

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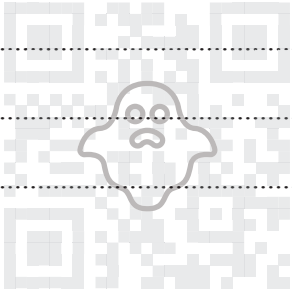
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- 5 Georgio has designed two new uniforms X and Y for the employees of an airline company. A random sample of 11 employees are each asked to assess each of the two uniforms for practicality and appearance, and to give a total score out of 100. The scores are given in the table.

Employee	A	B	C	D	E	F	G	H	I	J	K
Uniform X	82	74	42	59	60	73	94	98	62	36	50
Uniform Y	78	75	63	56	67	82	99	90	72	48	61

- (a) Give a reason why a Wilcoxon signed-rank test may be more appropriate than a t -test for investigating whether there is any evidence of a preference for one of the uniforms. [1]

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- (b) Carry out a Wilcoxon matched-pairs signed-rank test at the 10% significance level. [7]

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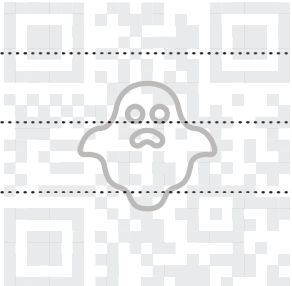
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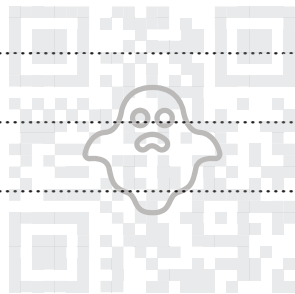
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6 Tanji has a bag containing 4 red balls and 2 blue balls. He selects 3 balls at random from the bag, without replacement. The number of red balls selected by Tanji is denoted by X .

(a) Find the probability generating function $G_X(t)$ of X . [2]

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Tanji also has two coins, each biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$. He throws the two coins at the same time. The number of heads obtained is denoted by Y .

(b) Find the probability generating function $G_Y(t)$ of Y . [2]

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The random variable Z is the sum of the number of red balls selected by Tanji and the number of heads obtained.

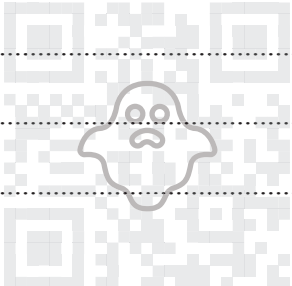
(c) Find the probability generating function of Z , expressing your answer as a polynomial. [3]

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(d) Use the probability generating function of Z to find $E(Z)$ and $\text{Var}(Z)$. [5]

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