1 A particle P of mass $1\,\text{kg}$ is moving along a straight line against a resistive force of magnitude $\dfrac{10\sqrt{v}}{(t+1)^2}\,\text{N}$, where $v\,\text{m s}^{-1}$ is the speed of P at time $t\,\text{s}$. When $t = 0$, $v = 25$.

Find an expression for v in terms of t. [5]

2   A hollow hemispherical bowl of radius a has a smooth inner surface and is fixed with its axis vertical. A particle P of mass m moves in horizontal circles on the inner surface of the bowl, at a height x above the lowest point of the bowl. The speed of P is $\sqrt{\frac{8}{3}ga}$.

Find x in terms of a.   [6]
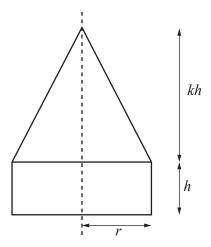
3 One end of a light elastic string, of natural length a and modulus of elasticity kmg, is attached to a fixed point A. The other end of the string is attached to a particle P of mass $4m$. The particle P hangs in equilibrium a distance x vertically below A.

(a) Show that $k = \dfrac{4a}{x-a}$.   [1]

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An additional particle, of mass $2m$, is now attached to P and the combined particle is released from rest at the original equilibrium position of P. When the combined particle has descended a distance $\frac{1}{3}a$, its speed is $\frac{1}{3}\sqrt{ga}$.

(b) Find x in terms of a.   [6]

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A uniform solid circular cone has vertical height kh and radius r. A uniform solid cylinder has height h and radius r. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram, which shows a cross-section). The cone and the cylinder are made of the same material.
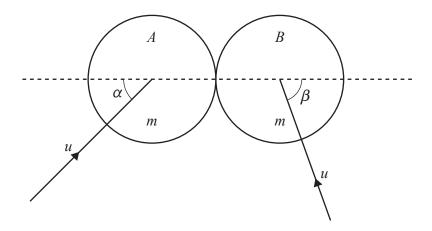
(a) Show that the distance of the centre of mass of the combined solid from the base of the cylinder is $\dfrac{h(k^2 + 4k + 6)}{4(3+k)}$. [4]

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The solid is placed on a plane that is inclined to the horizontal at an angle θ. The base of the cylinder is in contact with the plane. The plane is sufficiently rough to prevent sliding. It is given that $3h = 2r$ and that the solid is on the point of toppling when $\tan\theta = \frac{4}{3}$.

(b) Find the value of k. [3]

5 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle completes vertical circles with centre O. The points A and B are on the path of P, both on the same side of the vertical through O. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O.

The speed of P when it is at A is u and the speed of P when it is at B is $\sqrt{ag}$. The tensions in the string at A and B are T_A and T_B respectively. It is given that $T_A = 7T_B$.

Find the value of θ and find an expression for u in terms of a and g. [8]

6

Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision, A's direction of motion makes an angle α with the line of centres, and B's direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$ and $2\cos\beta = \cos\alpha$.

(a) Show that the direction of motion of A after the collision is perpendicular to the line of centres.

[4]

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The total kinetic energy of the spheres after the collision is $\frac{3}{4}mu^2$.

(b) Find the value of α. [4]

7   A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is $100\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. The two times at which P's height above the plane is H m differ by $10\,\mathrm{s}$.

   (a) Find the value of H.   [5]

(b) Find the magnitude and direction of the velocity of P one second before it strikes the plane. [4]