Pearson Edexcel AS Further Mathematics 8FM0

FPure2 – 1 Groups

Time allowed: 45 minutes

School: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	8	
2	10	
3	7	
4	8	
Total:	33	

How I can achieve better:

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1. The set $G = \{1, 3, 7, 9, 11, 13, 17, 19\}$ under the binary operation of multiplication modulo 20 forms a group.

   (a) Find the inverse of each element of G. [3]

   (b) Find the order of each element of G. [3]

   (c) Find a subgroup of G of order 4 [1]

   (d) Explain how the subgroup you found in part (c) satisfies Lagrange's theorem. [1]

   Total: 8

(Q1 continued . . .)

2. Figure below shows an equilateral triangle ABC. The lines x, y and z and their point of intersection, O, are fixed in the plane. The triangle ABC is transformed about these fixed lines and the fixed point O. The lines x, y and z each pass through a vertex of the triangle and the midpoint of the opposite side.
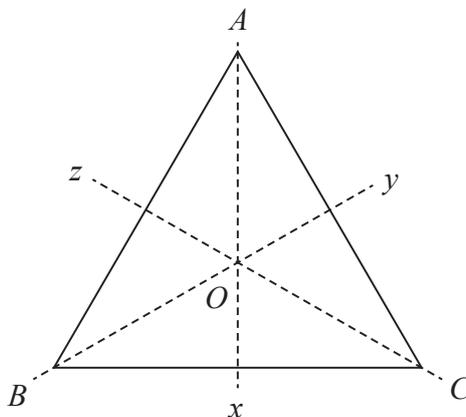
The transformations I, X, Y, Z, R_1 and R_2 of the plane containing triangle ABC are defined as follows:

- I: Do nothing

- X : Reflect in the line x

- Y: Reflect in the line y

- Z : Reflect in the line z

- R_1 : Rotate 120° anticlockwise about O

- R_2 : Rotate 240° anticlockwise about O

The operation * is defined as 'followed by' on the set $T = \{I, X, Y, Z, R_1, R_2\}$.

For example, X^*Y means a reflection in the line x followed by a reflection in the line y.

(a)   i. Copy and complete the Cayley table below. [6]

	*	I	X	Y	Z	R_1	R_2
	I						
	X		I			Z	
	Y						
First transformation	Z						
	R_1		Y				
	R_2						

(Column header "Second transformation" spans I, X, Y, Z, R_1, R_2; row label "First transformation" spans the rows.)

Given that the associative law is satisfied,

  ii. show that T is a group under the operation *.

(b) Show that the element R_2 has order 3 . [2]

(c) Explain why T is not a cyclic group. [1]

(d) Write down the elements of a subgroup of T that has order 3. [1]

Total: 10

(Q2 continued . . .)

(Q2 continued . . .)

3. The set $\{e, p, q, r, s\}$ forms a group, A, under the operation *.

   Given that e is the identity element and that

$$p^*p = s \quad s^*s = r \quad p^*p^*p = q$$

   (a) show that [2]

      i. $p^*q = r$

      ii. $s^*p = q$

   (b) Hence copy and complete the Cayley table below. [2]

*	e	p	q	r	s
e					
p					
q					
r					
s					

   (c) Use your table to find $p^*q^*r^*s$ [1]

   A student states that there is a subgroup of A of order 3.

   (d) Comment on the validity of this statement, giving a reason for your answer. [2]

   Total: 7

(Q3 continued . . .)

4. (a) The set $G = \{1, 3, 4, 9, 10, 12\}$ forms a group under the operation of multiplication modulo 13.

    i. Copy and complete the Cayley table below for this group. [3]

    ii. Find a subgroup of ($G, \times_{13}$) of order 3 . [1]

    iii. Explain why there can be no subgroup of ($G, \times_{13}$) of order 4. [1]

  (b) Determine whether the set $\{1, 3, 6, 9, 12\}$ under the operation of multiplication modulo 15 forms a group. [3]

$\times_{13}$	1	3	4	9	10	12
1		3				
3			12			
4						
9			10			
10	10					
12					3	

Total: 8

(Q4 continued ...)