Pearson Edexcel AS Further Mathematics 8FM0

Decision 2 − 2 Flows in Networks

Time allowed: 45 minutes

School: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	9	
2	13	
3	10	
4	10	
Total:	42	

How I can achieve better:

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1. Figure below shows a capacitated, directed network of pipes.
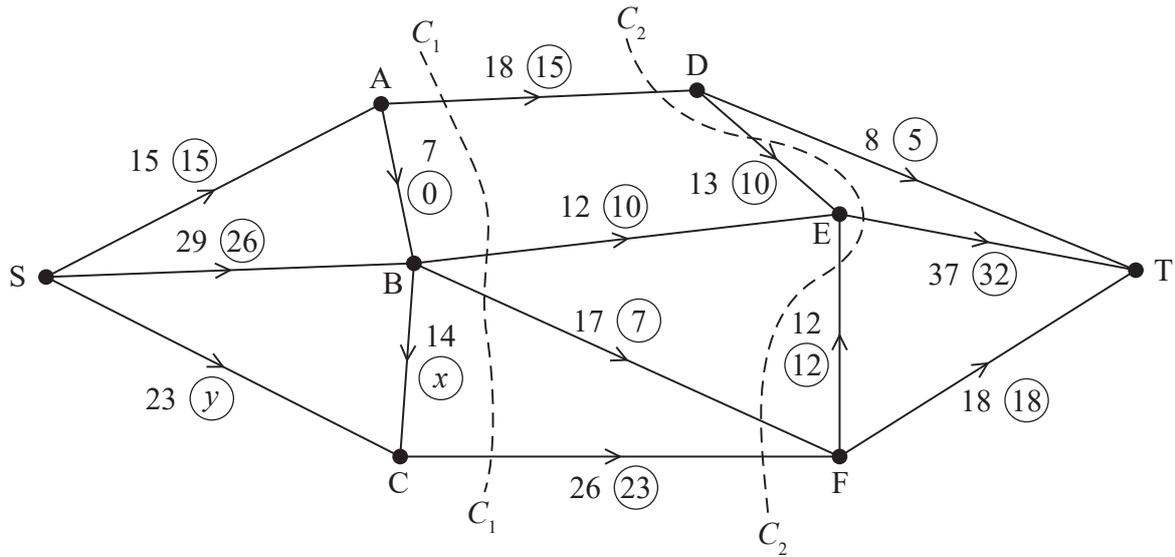
The number on each arc represents the capacity of the corresponding pipe. The numbers in circles represent a feasible flow from S to T.

(a)   i. Find the value of x.   [2]

     ii. Find the value of y.

(b) List the saturated arcs.   [1]

Two cuts, C_1 and C_2, are shown above.

(c) Find the capacity of   [2]

     i. C_1

    ii. C_2

(d) Write down a flow–augmenting route, using the arc CF , that increases the flow by two units.   [1]

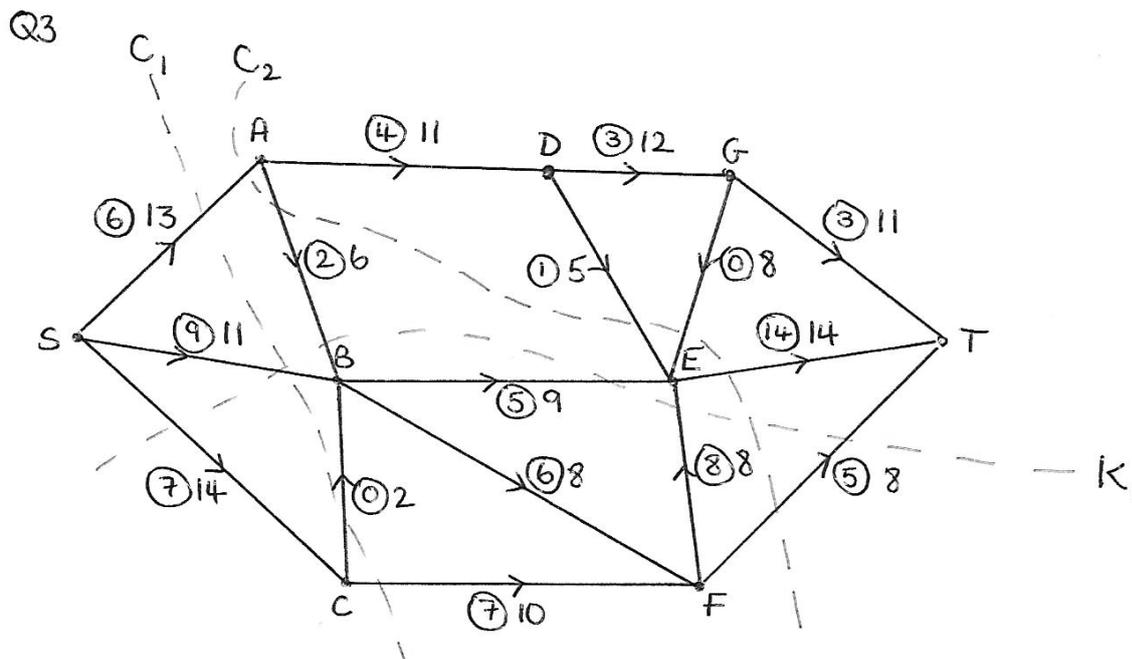Given that the flow through the network is increased by two units using the route found in (d),

(e) prove that this new flow is maximal.   [3]

Total: 9

(Q1 continued . . .)

2. Figure below shows a capacitated, directed network.

The number on each arc is the capacity of that arc. The numbers in circles represent an initial flow from S to T.

(a) Explain why the dotted line marked K_1 in Figure 1 is not a cut. [1]

Two cuts C_1 and C_2 are shown above.

(b) Write down [3]

    i. the capacity of each of the two cuts C_1 and C_2

    ii. the value of the initial flow.

(c) Complete the initialisation of the labelling procedure on Diagram 1 in the answer book by [2] entering values along arcs SA, AD, DE, GE and GT.

(d) Hence use the labelling procedure to find a maximal flow pattern for the network. You must [4]

    i. list each flow - augmenting path you use, together with its flow,

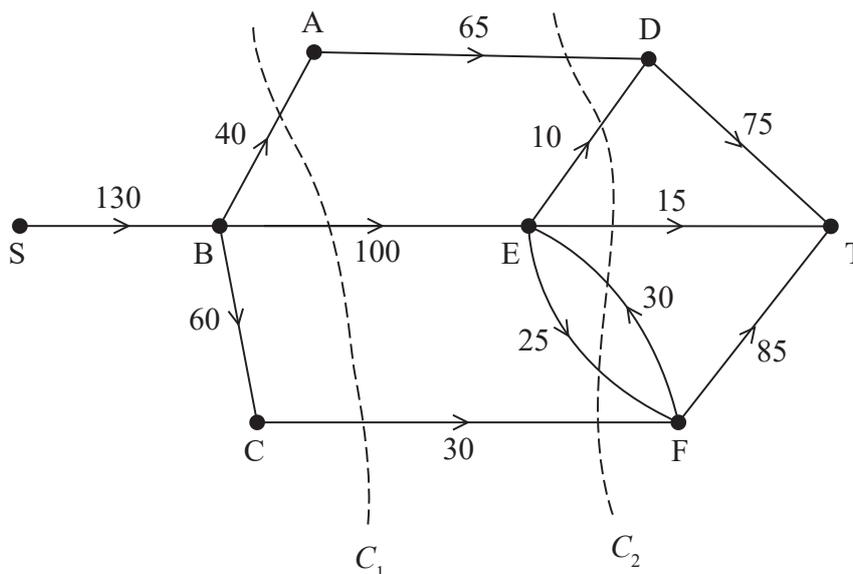   ii. show your maximal flow pattern on Diagram 2 in the answer book.

(e)   i. State the value of the maximal flow, [3]

    ii. prove that your flow is maximal.

Total: 13

(Q2 continued . . .)

3. Figure below models the flow of fluid through a system of pipes from a source, S, to a sink, T.

The weights on the arcs show the capacities of the corresponding pipes in litres per minute. Two cuts $C1$ and $C2$ are shown.

(a) Find the capacity of [2]

    i. cut $C1$,

  ii. cut $C2$.

(b) Using only the capacities of cuts $C1$ and $C2$ state what can be deduced about the maximum [1] possible flow through the system.

(c) On Diagram 1 in the answer book, show how a flow of 120 litres per minute from S to T [2] can be achieved. You do not need to apply the labelling procedure to find this flow.

(d) Prove that 120 litres per minute is the maximum possible flow through the system. [2]

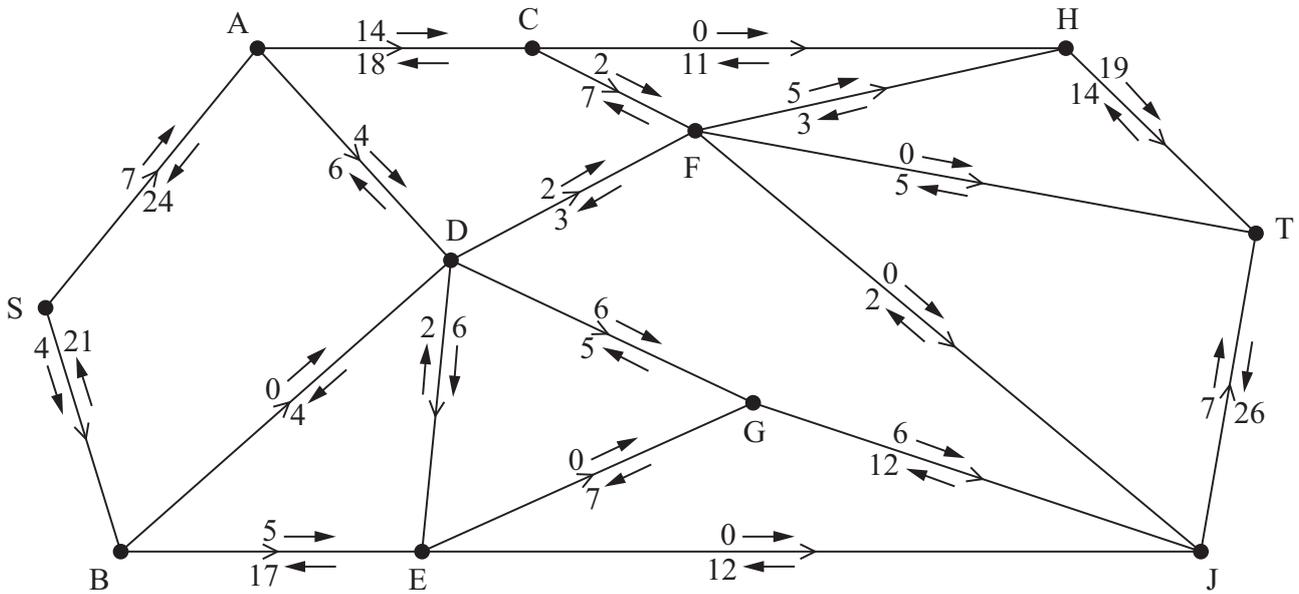A new pipe is planned from S to A. Let the capacity of this pipe be x litres per minute.

(e) Find, in terms of x where necessary, the maximum possible flow through the new system. [3]

Total: 10

(Q3 continued . . .)

4. Alexa is monitoring a system of pipes through which fluid can flow from the source, S, to the sink, T.

Currently, fluid is flowing through the system from S to T. Alexa initialises the labelling procedure for this system, and the excess capacities and potential backflows are shown on the arrows either side of each arc, as shown above.

(a) State the value of the initial flow. [1]

(b) Explain why arcs DF and DG can never both be full to capacity. [1]

(c) Obtain the capacity of the cut that passes through the arcs AC, AD, BD, DE, EG and EJ . [1]

(d) Use the labelling procedure to find a maximum flow through the network. You must list each flow–augmenting route you use, together with its flow. [3]

(e) Use your answers to part (d) to find a maximum flow pattern for this system of pipes and draw it on Diagram 1 in the answer book. [1]

(f) Prove that the answer to part (e) is optimal. [3]

Total: 10

(Q4 continued . . .)