

Pearson Edexcel AS Further Mathematics 8FM0

Core Pure – 3 Matrices

Time allowed: 45 minutes

School: www.CasperYC.club

Name:

Teacher:

How I can achieve better:

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Question	Points	Score
1	5	
2	10	
3	6	
4	6	
5	16	
6	13	
7	12	
Total:	68	

Last updated: February 3, 2026



5. (a)

$$\mathbf{A} = \begin{pmatrix} 2 & a \\ a - 4 & b \end{pmatrix}$$

where a and b are non-zero constants. Given that the matrix \mathbf{A} is self-inverse,

- i. determine the value of b and the possible values for a . [5]

The matrix \mathbf{A} represents a linear transformation M . Using the smaller value of a from part (a),

- ii. show that the invariant points of the linear transformation M form a line, stating the equation of this line. [3]

(b)

$$\mathbf{P} = \begin{pmatrix} p & 2p \\ -1 & 3p \end{pmatrix}$$

where p is a positive constant.

The matrix \mathbf{P} represents a linear transformation U .

The triangle T has vertices at the points with coordinates $(1, 2)$, $(3, 2)$ and $(2, 5)$.

The area of the image of T under the linear transformation U is 15.

- i. Determine the value of p . [4]

The transformation V consists of a stretch scale factor 3 parallel to the x -axis with the y -axis invariant followed by a stretch scale factor -2 parallel to the y -axis with the x -axis invariant. The transformation V is represented by the matrix \mathbf{Q} .

- ii. Write down the matrix \mathbf{Q} . [2]

- iii. Given that U followed by V is the transformation W , which is represented by the matrix \mathbf{R} , find the matrix \mathbf{R} . [2]

Total: 16



6. The population of the Zebu cattle in a particular country is modelled by two sub-populations, adults and juveniles. In this model, the only factors affecting the population of the Zebu are the birth and survival rates of the population.

Data recorded in the years preceding 2018 was used to suggest the annual birth and survival rates. The results are shown in the table below, with values to 2 significant figures. It is assumed that these rates will remain the same in future years.

	Birth rate	Survival Rate
Adult population	0.23	0.97
Juvenile population	0	0.87

It is also assumed that $\frac{1}{3}$ of the surviving juvenile population become adults each year.

Let A_n and J_n be these respective sub-populations, in millions of adults and juveniles, n years after 1st January 2018. Then the adult population in year $n + 1$ satisfies the equation

$$A_{n+1} = 0.97A_n + \frac{1}{3}(0.87)J_n = 0.97A_n + 0.29J_n.$$

- (a) Form the corresponding equation for the juvenile population in year $n + 1$ under this model, justifying your values. [2]

The total population on 1st January 2018 was estimated, to 2 significant figures, as 1.5 million Zebu, with 1.2 million of these being adults.

- (b) Find the value of p and the matrix \mathbf{M} such that the population of Zebu can be modelled by the system [3]

$$\begin{pmatrix} A_0 \\ J_0 \end{pmatrix} = \begin{pmatrix} 1.2 \\ p \end{pmatrix} \quad \begin{pmatrix} A_{n+1} \\ J_{n+1} \end{pmatrix} = \mathbf{M} \begin{pmatrix} A_n \\ J_n \end{pmatrix}$$

giving p to 2 significant figures and each entry of \mathbf{M} to 2 decimal places.

- (c) Using the model formed in part (b), find, to 3 significant figures, [5]
- the total Zebu population that was present on 1st January 2017,
 - the predicted juvenile Zebu population on 1st January 2025.



7. The population of chimpanzees in a particular country consists of juveniles and adults. Juvenile chimpanzees do not reproduce.

In a study, the numbers of juvenile and adult chimpanzees were estimated at the start of each year. A model for the population satisfies the matrix system

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix} \begin{pmatrix} J_n \\ A_n \end{pmatrix} \quad n = 0, 1, 2, \dots$$

where a is a constant, and J_n and A_n , are the respective numbers of juvenile and adult chimpanzees n years after the start of the study.

- (a) Interpret the meaning of the constant a in the context of the model. [1]

At the start of the study, the total number of chimpanzees in the country was estimated to be 64000.

According to the model, after one year the number of juvenile chimpanzees is 15360 and the number of adult chimpanzees is 43008.

- (b) i. Find, in terms of a [3]

$$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1}$$

- ii. Hence, or otherwise, find the value of a . [3]

- iii. Calculate the change in the number of juvenile chimpanzees in the first year of the study, according to this model. [2]

Given that the number of juvenile chimpanzees is known to be in decline in the country,

- (c) comment on the short-term suitability of this model. [1]

A study of the population revealed that adult chimpanzees stop reproducing at the age of 40 years.

- (d) Refine the matrix system for the model to reflect this information, giving a reason for your answer. (There is no need to estimate any unknown values for the refined model, but any known values should be made clear.) [2]



