1 (a) (i) At a football club, season tickets are sold for seated areas and for standing areas.
The cost of season tickets are in the ratio seated : standing $=5: 3$.
The cost of a season ticket for the standing area is $\$ 45$.
Find the cost of a season ticket for the seated area.
\$
(ii) In 2021, the value of the team's players was $\$ 2.65$ million.

In 2022 this value has decreased by $12 \%$.
Find the value in 2022.
\$. $\qquad$ million
(iii) The number of people at a football match is 1455 .

This is $6.25 \%$ of the total number of people allowed in the stadium.
Find the total number of people allowed in the stadium.
(iv) The average attendance increased exponentially by $4 \%$ each year for the three years from 2016 to 2019.
In 2019 the average attendance was 1631.
Find the average attendance for 2016.
(b) Another club sells season tickets for individuals and for families.

In 2018, the number of season tickets sold is in the ratio family : individual $=2: 7$.
(i) The number of family season tickets sold is $x$.

Write an expression, in terms of $x$, for the number of individual season tickets sold.
(ii) In 2019, the number of family season tickets sold increases by 12 and the number of individual season tickets sold decreases by 26 .

Complete the table by writing expressions, in terms of $x$, for the number of tickets sold each year.

| Year | Family tickets | Individual tickets |
| :--- | :---: | :--- |
| 2018 | $x$ |  |
| 2019 |  |  |

(iii) In 2019, the number of individual season tickets sold is 3 times the number of family season tickets sold.

Write an equation in $x$ and solve it to find the number of family tickets sold in 2018.

$$
\begin{equation*}
x= \tag{4}
\end{equation*}
$$

2 All the lengths in this question are measured in centimetres.


## NOT TO <br> SCALE

The diagram shows a solid cuboid with a square base.
(a) The volume, $V \mathrm{~cm}^{3}$, of the cuboid is $V=x^{2}(9-x)$.

The table shows some values of $V$ for $0 \leqslant x \leqslant 9$.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| $V$ | 0 | 8 |  | 54 | 80 | 100 | 108 | 98 | 64 | 0 |

(i) Complete the table.
(ii) On the grid on the opposite page, draw the graph of $V=x^{2}(9-x)$ for $0 \leqslant x \leqslant 9$.
(iii) Find the values of $x$ when the volume of the cuboid is $44 \mathrm{~cm}^{3}$.

$$
\begin{equation*}
x= \tag{2}
\end{equation*}
$$

$\qquad$ or $x=$

(b) (i) Show that the total surface area of the cuboid is $\left(36 x-2 x^{2}\right) \mathrm{cm}^{2}$.
(ii) Find the surface area when the volume of the cuboid is a maximum.
$\qquad$ $\mathrm{cm}^{2}$

3 Kai and Ann carry out a survey on the distances travelled, in kilometres, by 200 cars.
Kai completes this frequency table for the data collected.

| Distance $(d \mathrm{~km})$ | $80<d \leqslant 100$ | $100<d \leqslant 150$ | $150<d \leqslant 200$ | $200<d \leqslant 300$ | $300<d \leqslant 400$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 7 | 33 | 76 | 52 | 32 |

(a) (i) Calculate an estimate of the mean.
(ii) Ann uses this frequency table for the same data.

There is a different interval for the final group.

| Distance $(d \mathrm{~km})$ | $80<d \leqslant 100$ | $100<d \leqslant 150$ | $150<d \leqslant 200$ | $200<d \leqslant 300$ | $300<d \leqslant 360$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 7 | 33 | 76 | 52 | 32 |

Without calculating an estimate of the mean for this data, find the difference between Ann's and Kai's estimate of the mean.
You must show all your working.
(iii) A histogram is drawn showing the information in Kai's frequency table.

The height of the block for the interval $200<d \leqslant 300$ is 2.6 cm .
Calculate the height of the block for each of the following intervals.

$$
\begin{aligned}
80 & <d \leqslant 100 \text {........................................... } \mathrm{cm} \\
150 & <d \leqslant 200 \text {............................................ } \mathrm{cm} \\
300 & <d \leqslant 400 \text {............................................. } \mathrm{cm}
\end{aligned}
$$

(b) One car is picked at random.

Find the probability that the car has travelled more than 300 km .
(c) Two of the 200 cars are picked at random.

Find the probability that
(i) both cars have travelled 150 km or less,
(ii) one car has travelled more than 200 km and the other car has travelled 100 km or less.

(a) Describe fully the single transformation that maps
(i) shape $A$ onto shape $B$,
$\qquad$
$\qquad$
(ii) shape $A$ onto shape $C$,
$\qquad$
$\qquad$
(iii) shape $A$ onto shape $D$.
$\qquad$
$\qquad$
(b) On the grid, draw the image of shape $A$ after a reflection in the line $y=x+8$.

5 (a) The diagram shows the speed-time graph for part of a journey for two vehicles, a car and a bus.

(i) Calculate the acceleration of the car during the first 18 seconds.
$\qquad$
(ii) In the first 40 seconds the car travelled 134 m more than the bus.

Calculate the constant speed, $v$, of the bus.

$$
v=. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~ m / s ~[4] ~
$$

(b) A train takes 10 minutes 30 seconds to travel 16240 m .

Calculate the average speed of the train.
Give your answer in kilometres per hour.

6 (a) Solve.

$$
4 x+15=9
$$

$$
\begin{equation*}
x= \tag{2}
\end{equation*}
$$

(b) Factorise.

$$
a^{2}-9
$$

(c) Write as a single fraction in its simplest form.

$$
\frac{4 a}{5} \div \frac{3 a d}{10 c}
$$

(d) $5^{n}+5^{n}+5^{n}+5^{n}+5^{n}=5^{m}$

Find an expression for $m$ in terms of $n$.
$m=$
(e) Solve by factorisation.

$$
4 x^{2}+8 x-5=0
$$

$$
x=
$$

$\qquad$ or $x=$
(f) (i) $y$ is directly proportional to $(x+3)^{3}$. When $x=2, y=13.5$.

Find $x$ when $y=108$.

$$
x=
$$

(ii) $g$ is inversely proportional to the square of $d$. When $d$ is halved, the value of $g$ is multiplied by a factor $n$.

Find $n$.

$$
n=
$$

(g) Expand and simplify.

$$
(2 x+3)(x-1)(x+3)
$$

(h) Find the derivative, $\frac{\mathrm{d} y}{\mathrm{~d} x}$, of $y=3 x^{2}+4 x-1$.

7 (a)


NOT TO
SCALE
(i) Calculate angle $Q P R$.

Angle $Q P R=$
(ii) Find the shortest distance from $Q$ to $P R$.
(b) The diagram shows a cuboid.


NOT TO
SCALE
(i) Calculate the length $A G$.
(ii) Calculate the angle between $A G$ and the base $A B C D$.
(c)


NOT TO
SCALE

The diagram shows the positions of a lighthouse, $L$, and two ships, $K$ and $M$.
The bearing of $L$ from $K$ is $155^{\circ}$ and $K L=112 \mathrm{~km}$.
The bearing of $K$ from $M$ is $010^{\circ}$ and angle $K M L=96^{\circ}$.
Find the bearing and distance of ship $M$ from the lighthouse, $L$.
$\qquad$
$8 \quad A B$ is a line with midpoint $M$.
$A$ is the point $(2,3)$ and $M$ is the point $(12,7)$.
(a) Find the coordinates of $B$.
$\qquad$
(b) Show that the equation of the perpendicular bisector of $A B$ is $2 y+5 x=74$.
(c) The perpendicular bisector of $A B$ passes through the point $N$.

The point $N$ has coordinates $(2, n)$.
Find the value of $n$.

$$
n=
$$

(d) Points $A, M$ and $N$ form a triangle.

Find the area of the triangle.


(a) On the diagram, sketch the graph of $y=\sin x$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$.
(b) Solve the equation $5 \sin x+4=0$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$.

$$
x=
$$

$\qquad$ or $x=$

10 (a) The lengths of the sides of a triangle are $11.4 \mathrm{~cm}, 14.8 \mathrm{~cm}$ and 15.7 cm , all correct to 1 decimal place.

Calculate the upper bound of the perimeter of the triangle.
(b)


NOT TO
SCALE

The diagram shows a circle, radius 15.6 cm .
The angle of the minor sector is $150^{\circ}$.
Calculate the area of the minor sector.
(c)


NOT TO
SCALE

The diagram shows a circle, radius $r \mathrm{~cm}$ and minor sector angle $x^{\circ}$.
The perimeter of the major sector is three times the perimeter of the minor sector.
Show that $x=\frac{90(\pi-2)}{\pi}$.

11 (a) $\quad\left|\binom{9 m}{40 m}\right|=\frac{205}{2}$
Find the two possible values of $m$.
$m=$ $\qquad$ or
(b)


> NOT TO
> SCALE
$O A B C$ is a parallelogram.
$\overrightarrow{O A}=\mathbf{a}$ and $\overrightarrow{O C}=\mathbf{c}$.
$P$ is the point on $C B$ such that $C P: P B=3: 1$.
(i) Find, in terms of $\mathbf{a}$ and/or $\mathbf{c}$, in their simplest form,
(a) $\overrightarrow{A C}$,

$$
\begin{equation*}
\overrightarrow{A C}= \tag{1}
\end{equation*}
$$

(b) $\overrightarrow{C P}$,

$$
\overrightarrow{C P}=
$$

(c) $\overrightarrow{O P}$.

$$
\overrightarrow{O P}=
$$

(ii) $O P$ and $A B$ are extended to meet at $Q$.

Find the position vector of $Q$.

