# AMC10 Past Paper Collections

Year 2020 — 2000

Updated on: December 26, 2020



Comments and suggestions to 89272376@QQ.com

# Answer Keys

| | 2000 | 2001 | 2002 | | 2003 | | 2004 | | 2005 | | 2006 | | 2007 | | 2008 | | 2009 | | 2010 | | 2011 | | 2012 | | 2013 | | 2014 | | 2015 | | 2016 | | 2017 | | 2018 | | 2019 | | 2020 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | |
| Q1 | E | E | D | E | D | C | A | C | D | A | A | C | C | E | D | E | E | B | D | C | D | C | D | C | C | C | C | C | C | C | B | D | C | B | B | A | C | D | E | D | Q1 |
| Q2 | A | C | C | C | B | D | B | B | C | D | C | A | A | E | A | B | A | C | B | C | E | E | E | E | B | A | C | E | D | B | C | B | D | C | A | D | A | E | C | E | Q2 |
| Q3 | B | E | B | A | D | B | E | A | B | D | B | A | D | B | A | D | C | C | D | C | D | A | E | B | E | C | E | E | D | A | C | D | B | E | E | B | D | B | A | E | Q3 |
| Q4 | D | E | E | D | A | A | D | B | B | D | E | D | A | D | C | C | A | C | B | C | A | C | C | A | C | D | B | B | B | C | B | B | B | D | E | B | B | A | E | D | Q4 |
| Q5 | B | D | C | E | B | C | C | D | A | C | D | B | B | D | B | A | E | D | E | B | C | E | B | D | B | B | C | A | E | B | D | D | C | D | D | D | D | E | C | B | Q5 |
| Q6 | C | E | A | B | C | D | E | C | B | B | B | D | A | D | D | C | A | D | C | B | C | A | D | A | D | C | A | C | B | E | D | B | B | B | B | D | C | C | B | B | Q6 |
| Q7 | B | C | A | E | B | B | C | A | B | B | A | A | D | E | E | C | C | C | C | D | B | B | C | D | C | B | B | A | B | A | D | C | A | C | E | D | C | B | C | A | Q7 |
| Q8 | D | B | A | D | E | B | B | A | C | A | E | B | D | D | A | C | B | B | D | E | C | B | D | B | C | B | D | E | B | E | C | A | B | C | C | C | C | B | B | D | Q8 |
| Q9 | C | B | B | D | A | B | B | B | B | D | C | B | E | D | B | A | B | A | E | D | A | D | D | A | B | D | C | A | D | B | D | C | C | D | E | D | B | A | B | D | Q9 |
| Q10 | D | D | A | C | E | C | D | D | A | A | E | A | E | A | E | A | B | E | E | C | B | B | C | D | E | C | B | C | C | C | B | D | B | E | A | E | C | A | B | C | Q10 |
| Q11 | C | C | B | B | E | A | C | C | B | E | C | C | C | C | D | B | D | A | D | A | B | D | D | A | A | B | C | C | C | B | D | B | D | D | E | C | C | A | C | D | Q11 |
| Q12 | C | D | B | E | A | C | C | A | B | E | C | E | D | D | C | A | C | A | C | D | A | A | A | B | C | B | C | C | C | A | A | D | E | A | C | C | E | C | C | D | Q12 |
| Q13 | B | E | B | D | A | E | D | B | E | C | D | E | B | D | D | B | E | C | A | C | A | D | C | B | B | E | C | B | C | E | B | D | D | C | D | C | D | A | B | B | Q13 |
| Q14 | C | A | B | B | A | D | A | C | E | C | B | D | A | C | D | B | A | D | C | B | B | C | B | D | D | E | D | D | C | D | C | D | D | D | A | D | D | C | D | D | Q14 |
| Q15 | E | C | E | E | C | E | D | A | E | D | D | C | B | D | D | A | E | E | D | C | C | E | B | D | D | B | C | A | B | B | A | C | C | E | D | A | E | A | E | D | Q15 |
| Q16 | B | D | B | D | C | E | D | D | D | D | D | E | E | C | B | A | D | B | B | B | B | A | C | A | E | B | E | B | B | C | D | E | B | A | D | E | A | A | B | A | Q16 |
| Q17 | D | C | D | C | B | B | C | B | D | B | A | D | D | D | B | B | C | C | A | B | C | C | C | C | B | E | D | D | D | B | A | D | D | B | C | B | D | C | E | C | Q17 |
| Q18 | C | D | D | D | B | D | A | E | A | D | C | E | C | B | B | B | D | D | B | E | C | E | E | C | B | D | B | E | E | D | C | E | D | D | D | D | D | C | C | B | Q18 |
| Q19 | D | D | E | C | C | E | C | C | D | B | C | A | C | C | C | E | B | A | E | B | E | A | D | C | C | D | A | D | D | C | E | D | C | E | E | E | B | C | E | A | Q19 |
| Q20 | C | B | D | B | E | D | D | D | A | C | E | E | D | C | D | B | D | B | D | D | D | C | A | A | C | B | D | C | B | A | B | C | D | B | B | B | B | E | D | B | Q20 |
| Q21 | B | B | D | B | D | C | B | A | B | A | E | C | C | B | A | C | C | B | A | E | D | B | C | A | D | C | E | B | C | D | D | B | D | D | E | C | D | B | C | B | Q21 |
| Q22 | C | D | C | B | B | B | D | D | D | C | C | D | D | B | D | C | D | B | A | C | C | A | A | B | B | C | E | B | A | D | D | A | E | D | D | C | B | B | A | D | Q22 |
| Q23 | E | D | D | D | C | D | D | B | C | C | B | D | B | E | B | B | E | C | A | D | C | D | B | D | D | B | C | E | C | B | A | C | B | C | D | B | C | C | A | C | Q23 |
| Q24 | B | B | A | D | E | E | D | B | B | E | B | B | B | C | D | C | C | A | A | E | D | B | E | B | E | A | A | B | B | D | E | D | C | C | D | C | B | C | C | C | Q24 |
| Q25 | A | B | C | A | B | B | B | B | D | C | C | B | D | A | C | B | B | B | B | B | C | D | D | E | A | E | B | C | A | B | A | A | A | E | D | C | D | C | A | A | Q25 |

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**Q1**. What value of $x$ satisfies

$$x - \frac{3}{4} = \frac{5}{12} - \frac{1}{3}?$$

**A)** $-\frac{2}{3}$ **B)** $\frac{7}{36}$ **C)** $\frac{7}{12}$ **D)** $\frac{2}{3}$ **E)** $\frac{5}{6}$

**Q2**. The numbers $3, 5, 7, a$, and $b$ have an average (arithmetic mean) of 15. What is the average of $a$ and $b$?

**A)** 0 **B)** 15 **C)** 30 **D)** 45 **E)** 60

**Q3**. Assuming $a \neq 3$, $b \neq 4$, and $c \neq 5$, what is the value in simplest form of the following expression?

$$\frac{a-3}{5-c} \cdot \frac{b-4}{3-a} \cdot \frac{c-5}{4-b}$$

**A)** $-1$ **B)** 1 **C)** $\frac{abc}{60}$ **D)** $\frac{1}{abc} - \frac{1}{60}$ **E)** $\frac{1}{60} - \frac{1}{abc}$

**Q4**. A driver travels for 2 hours at 60 miles per hour, during which her car gets 30 miles per gallon of gasoline. She is paid \$0.50 per mile, and her only expense is gasoline at \$2.00 per gallon. What is her net rate of pay, in dollars per hour, after this expense?

**A)** 20 **B)** 22 **C)** 24 **D)** 25 **E)** 26

**Q5**. What is the sum of all real numbers $x$ for which $|x^2 - 12x + 34| = 2$?

**A)** 12 **B)** 15 **C)** 18 **D)** 21 **E)** 25

**Q6**. How many 4-digit positive integers (that is, integers between 1000 and 9999, inclusive) having only even digits are divisible by 5?

**A)** 80 **B)** 100 **C)** 125 **D)** 200 **E)** 500

**Q7**. The 25 integers from $-10$ to 14, inclusive, can be arranged to form a 5-by-5 square in which the sum of the numbers in each row, the sum of the numbers in each column, and the sum of the numbers along each of the main diagonals are all the same. What is the value of this common sum?

**A)** 2 **B)** 5 **C)** 10 **D)** 25 **E)** 50

**Q8**. What is the value of

$$1 + 2 + 3 - 4 + 5 + 6 + 7 - 8 + \cdots + 197 + 198 + 199 - 200?$$

**A)** $9,800$ **B)** $9,900$ **C)** $10,000$ **D)** $10,100$ **E)** $10,200$

**Q9**. A single bench section at a school event can hold either 7 adults or 11 children. When $N$ bench sections are connected end to end, an equal number of adults and children seated together will occupy all the bench space. What is the least possible positive integer value of $N$?

**A)** 9 **B)** 18 **C)** 27 **D)** 36 **E)** 77

**Q10**. Seven cubes, whose volumes are 1, 8, 27, 64, 125, 216, and 343 cubic units, are stacked vertically to form a tower in which the volumes of the cubes decrease from bottom to top. Except for the bottom cube, the bottom face of each cube lies completely on top of the cube below it. What is the total surface area of the tower (including the bottom) in square units?

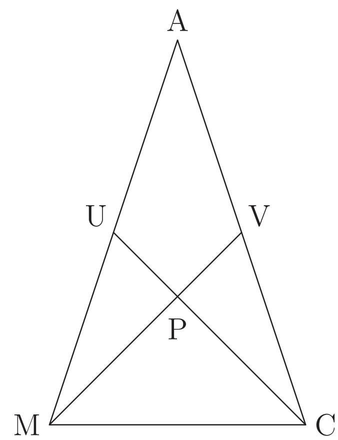
**A)** 644 **B)** 658 **C)** 664 **D)** 720 **E)** 749

**Q11**. What is the median of the following list of 4040 numbers?

$$1, 2, 3, \ldots, 2020, 1^2, 2^2, 3^2, \ldots, 2020^2$$

**A)** 1974.5 **B)** 1975.5 **C)** 1976.5 **D)** 1977.5 **E)** 1978.5

**Q12**. Triangle $AMC$ is isosceles with $AM = AC$. Medians $\overline{MV}$ and $\overline{CU}$ are perpendicular to each other, and $MV = CU = 12$. What is the area of $\triangle AMC$?




**A)** 48 **B)** 72 **C)** 96 **D)** 144 **E)** 192

**Q13**. A frog sitting at the point $(1, 2)$ begins a sequence of jumps, where each jump is parallel to one of the coordinate axes and has length 1, and the direction of each jump (up, down, right, or left) is chosen independently at random. The sequence ends when the frog reaches a side of the square with vertices $(0, 0), (0, 4), (4, 4)$, and $(4, 0)$. What is the probability that the sequence of jumps ends on a vertical side of the square?

**A)** $\frac{1}{2}$ **B)** $\frac{5}{8}$ **C)** $\frac{2}{3}$ **D)** $\frac{3}{4}$ **E)** $\frac{7}{8}$

**Q14**. Real numbers $x$ and $y$ satisfy $x + y = 4$ and $x \cdot y = -2$. What is the value of

$$x + \frac{x^3}{y^2} + \frac{y^3}{x^2} + y?$$

**A)** 360 **B)** 400 **C)** 420 **D)** 440 **E)** 480

**Q15**. A positive integer divisor of 12! is chosen at random. The probability that the divisor chosen is a perfect square can be expressed as $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. What is $m + n$?

**A)** 3 **B)** 5 **C)** 12 **D)** 18 **E)** 23

**Q16**. A point is chosen at random within the square in the coordinate plane whose vertices are $(0, 0), (2020, 0), (2020, 2020)$, and $(0, 2020)$. The probability that the point is within $d$ units of a lattice point is $\frac{1}{2}$. (A point $(x, y)$ is a lattice point if $x$ and $y$ are both integers.) What is $d$ to the nearest tenth?

**A)** 0.3 **B)** 0.4 **C)** 0.5 **D)** 0.6 **E)** 0.7

**Q17**. Define

$$\mathrm{P}(x) = (x - 1^2)(x - 2^2) \cdots (x - 100^2)$$

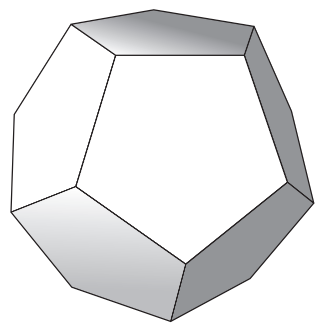
. How many integers $n$ are there such that $P(n) \le 0$?

**A)** 4900 **B)** 4950 **C)** 5000 **D)** 5050 **E)** 5100

**Q18**. Let $(a, b, c, d)$ be an ordered quadruple of not necessarily distinct integers, each one of them in the set $0, 1, 2, 3$. For how many such quadruples is it true that $a \cdot d - b \cdot c$ is odd? (For example, $(0, 3, 1, 1)$ is one such quadruple, because $0 \cdot 1 - 3 \cdot 1 = -3$ is odd.)

**A)** 48 **B)** 64 **C)** 96 **D)** 128 **E)** 192

**Q19**. As shown in the figure below, a regular dodecahedron (the polyhedron consisting of 12 congruent regular pentagonal faces) floats in space with two horizontal faces. Note that there is a ring of five slanted faces adjacent to the top face, and a ring of five slanted faces adjacent to the bottom face. How many ways are there to move from the top face to the bottom face via a sequence of adjacent faces so that each face is visited at most once and moves are not permitted from the bottom ring to the top ring?



**A)** 125 **B)** 250 **C)** 405 **D)** 640 **E)** 810

**Q20**. Quadrilateral $ABCD$ satisfies $\angle ABC = \angle ACD = 90^\circ, AC = 20$, and $CD = 30$. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at point $E$, and $AE = 5$. What is the area of quadrilateral $ABCD$?

**A)** 330 **B)** 340 **C)** 350 **D)** 360 **E)** 370

**Q21**. There exists a unique strictly increasing sequence of nonnegative integers $a_1 < a_2 < \& < a_k$ such that

$$\frac{2^{289}+1}{2^{17}+1} = 2^{a_1} + 2^{a_2} + \ldots + 2^{a_k}.$$

What is $k$?

**A)** 117 **B)** 136 **C)** 137 **D)** 273 **E)** 306

**Q22**. For how many positive integers $n \leq 1000$ is

$$\left\lfloor \frac{998}{n} \right\rfloor + \left\lfloor \frac{999}{n} \right\rfloor + \left\lfloor \frac{1000}{n} \right\rfloor$$

not divisible by 3? (Recall that $\lfloor x \rfloor$ is the greatest integer less than or equal to $x$.)

**A)** 22 **B)** 23 **C)** 24 **D)** 25 **E)** 26

**Q23**. Let $T$ be the triangle in the coordinate plane with vertices $(0,0), (4,0)$, and $(0,3)$. Consider the following five isometries (rigid transformations) of the plane: rotations of $90^\circ, 180^\circ$, and $270^\circ$ counterclockwise around the origin, reflection across the $x$-axis, and reflection across the $y$-axis. How many of the 125 sequences of three of these transformations (not necessarily distinct) will return $T$ to its original position? (For example, a $180^\circ$ rotation, followed by a reflection across the $x$-axis, followed by a reflection across the $y$-axis will return $T$ to its original position, but a $90^\circ$ rotation, followed by a reflection across the $x$-axis, followed by another reflection across the $x$-axis will not return $T$ to its original position.)

**A)** 12 **B)** 15 **C)** 17 **D)** 20 **E)** 25

**Q24**. Let $n$ be the least positive integer greater than 1000 for which

$$\gcd(63, n+120) = 21 \quad \text{and} \quad \gcd(n+63, 120) = 60.$$

What is the sum of the digits of $n$?

**A)** 12 **B)** 15 **C)** 18 **D)** 21 **E)** 24

**Q25**. Jason rolls three fair standard six-sided dice. Then he looks at the rolls and chooses a subset of the dice (possibly empty, possibly all three dice) to reroll. After rerolling, he wins if and only if the sum of the numbers face up on the three dice is exactly 7. Jason always plays to optimize his chances of winning. What is the probability that he chooses to reroll exactly two of the dice?

**A)** $\frac{7}{36}$ **B)** $\frac{5}{24}$ **C)** $\frac{2}{9}$ **D)** $\frac{17}{72}$ **E)** $\frac{1}{4}$

**Q1**. What is the value of

$$1-(-2)-3-(-4)-5-(-6)?$$

**A)** $-20$ **B)** $-3$ **C)** 3 **D)** 5 **E)** 21

**Q2**. Carl has 5 cubes each having side length 1, and Kate has 5 cubes each having side length 2. What is the total volume of these 10 cubes?

**A)** 24 **B)** 25 **C)** 28 **D)** 40 **E)** 45

**Q3**. The ratio of $w$ to $x$ is $4:3$, the ratio of $y$ to $z$ is $3:2$, and the ratio of $z$ to $x$ is $1:6$. What is the ratio of $w$ to $y$?

**A)** 4: 3 **B)** 3: 2 **C)** 8: 3 **D)** 4: 1 **E)** 16: 3

**Q4**. The acute angles of a right triangle are $a^\circ$ and $b^\circ$, where $a>b$ and both $a$ and $b$ are prime numbers. What is the least possible value of $b$?

**A)** 2 **B)** 3 **C)** 5 **D)** 7 **E)** 11

**Q5**. How many distinguishable arrangements are there of 1 brown tile, 1 purple tile, 2 green tiles, and 3 yellow tiles in a row from left to right? (Tiles of the same color are indistinguishable.)

**A)** 210 **B)** 420 **C)** 630 **D)** 840 **E)** 1050

**Q6**. Driving along a highway, Megan noticed that her odometer showed 15951 (miles). This number is a palindrome-it reads the same forward and backward. Then 2 hours later, the odometer displayed the next higher palindrome. What was her average speed, in miles per hour, during this 2-hour period?

**A)** 50 **B)** 55 **C)** 60 **D)** 65 **E)** 70

**Q7**. How many positive even multiples of 3 less than 2020 are perfect squares?

**A)** 7 **B)** 8 **C)** 9 **D)** 10 **E)** 12

**Q8**. Points $P$ and $Q$ lie in a plane with $PQ=8$. How many locations for point $R$ in this plane are there such that the triangle with vertices $P$, $Q$, and $R$ is a right triangle with area 12 square units?

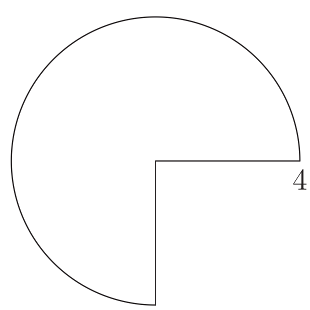
**A)** 2 **B)** 4 **C)** 6 **D)** 8 **E)** 12

**Q9**. How many ordered pairs of integers $(x,y)$ satisfy the equation

$$x^{2020}+y^2=2y?$$

**A)** 1 **B)** 2 **C)** 3 **D)** 4 **E)** infinitely many

**Q10**. A three-quarter sector of a circle of radius 4 inches together with its interior can be rolled up to form the lateral surface area of a right circular cone by taping together along the two radii shown. What is the volume of the cone in cubic inches?



**A)** $3\pi\sqrt{5}$ **B)** $4\pi\sqrt{3}$ **C)** $3\pi\sqrt{7}$ **D)** $6\pi\sqrt{3}$ **E)** $6\pi\sqrt{7}$

**Q11**. Ms. Carr asks her students to read any 5 of the 10 books on a reading list. Harold randomly selects 5 books from this list, and Betty does the same. What is the probability that there are exactly 2 books that they both select?

**A)** $\frac{1}{8}$ **B)** $\frac{5}{36}$ **C)** $\frac{14}{45}$ **D)** $\frac{25}{63}$ **E)** $\frac{1}{2}$

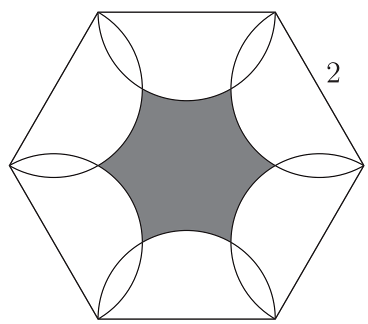
**Q12**. The decimal representation of $\frac{1}{20^{20}}$ consists of a string of zeros after the decimal point, followed by a 9 and then several more digits. How many zeros are in that initial string of zeros after the decimal point?

**A)** 23 **B)** 24 **C)** 25 **D)** 26 **E)** 27

**Q13**. Andy the Ant lives on a coordinate plane and is currently at $(-20, 20)$ facing east (that is, in the positive $x$-direction). Andy moves 1 unit and then turns 90° degrees left. From there, Andy moves 2 units (north) and then turns 90° degrees left. He then moves 3 units (west) and again turns 90° degrees left. Andy continues his progress, increasing his distance each time by 1 unit and always turning left. What is the location of the point at which Andy makes the 2020th left turn?

**A)** $(-1030, -994)$ **B)** $(-1030, -990)$ **C)** $(-1026, -994)$ **D)** $(-1026, -990)$ **E)** $(-1022, -994)$

**Q14**. As shown in the figure below, six semicircles lie in the interior of a regular hexagon with side length 2 so that the diameters of the semicircles coincide with the sides of the hexagon. What is the area of the shaded region — inside the hexagon but outside all of the semicircles?




**A)** $6\sqrt{3} - 3\pi$ **B)** $\frac{9\sqrt{3}}{2} - 2\pi$ **C)** $\frac{3\sqrt{3}}{2} - \frac{\pi}{3}$ **D)** $3\sqrt{3} - \pi$ **E)** $\frac{9\sqrt{3}}{2} - \pi$

**Q15**. Steve wrote the digits 1, 2, 3, 4, and 5 in order repeatedly from left to right, forming a list of $10,000$ digits, beginning 123451234512.... He then erased every third digit from his list (that is, the 3rd, 6th, 9th, ... digits from the left), then erased every fourth digit from the resulting list (that is, the 4th, 8th, 12th, ... digits from the left in what remained), and then erased every fifth digit from what remained at that point. What is the sum of the three digits that were then in the positions $2019, 2020, 2021$?

**A)** 7 **B)** 9 **C)** 10 **D)** 11 **E)** 12

**Q16**. Bela and Jenn play the following game on the closed interval $[0, n]$ of the real number line, where $n$ is a fixed integer greater than 4. They take turns playing, with Bela going first. At his first turn, Bela chooses any real number in the interval $[0, n]$. Thereafter, the player whose turn it is chooses a real number that is more than one unit away from all numbers previously chosen by either player. A player unable to choose such a number loses. Using optimal strategy, which player will win the game?

**A)** Bela will always win.
**B)** Jenn will always win.
**C)** Bela will win if and only if $n$ is odd.
**D)** Jenn will win if and only if $n$ is odd.
**E)** Jenn will win if and only if $n > 8$.

**Q17**. There are 10 people standing equally spaced around a circle. Each person knows exactly 3 of the other 9 people: the 2 people standing next to her or him, as well as the person directly across the circle. How many ways are there for the 10 people to split up into 5 pairs so that the members of each pair know each other?

**A)** 11 **B)** 12 **C)** 13 **D)** 14 **E)** 15

**Q18**. An urn contains one red ball and one blue ball. A box of extra red and blue balls lie nearby. George performs the following operation four times: he draws a ball from the urn at random and then takes a ball of the same color from the box and returns those two matching balls to the urn. After the four iterations the urn contains six balls. What is the probability that the urn contains three balls of each color?

**A)** $\frac{1}{6}$ **B)** $\frac{1}{5}$ **C)** $\frac{1}{4}$ **D)** $\frac{1}{3}$ **E)** $\frac{1}{2}$

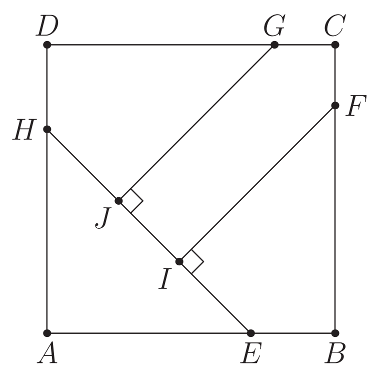
**Q19**. In a certain card game, a player is dealt a hand of 10 cards from a deck of 52 distinct cards. The number of distinct (unordered) hands that can be dealt to the player can be written as `158A00A4AA0`. What is the digit $A$?

**A)** 2 **B)** 3 **C)** 4 **D)** 6 **E)** 7

**Q20**. Let $B$ be a right rectangular prism (box) with edges lengths 1, 3, and 4, together with its interior. For real $r \geq 0$, let $S(r)$ be the set of points in 3-dimensional space that lie within a distance $r$ of some point in $B$. The volume of $S(r)$ can be expressed as $ar^3 + br^2 + cr + d$, where $a$, $b$, $c$, and $d$ are positive real numbers. What is $\frac{bc}{ad}$?

**A)** 6 **B)** 19 **C)** 24 **D)** 26 **E)** 38

**Q21**. In square $ABCD$, points $E$ and $H$ lie on $\overline{AB}$ and $\overline{DA}$, respectively, so that $AE = AH$. Points $F$ and $G$ lie on $\overline{BC}$ and $\overline{CD}$, respectively, and points $I$ and $J$ lie on $\overline{EH}$ so that $\overline{FI} \perp \overline{EH}$ and $\overline{GJ} \perp \overline{EH}$. See the figure below. Triangle $AEH$, quadrilateral $BFIE$, quadrilateral $DHJG$, and pentagon $FCGJI$ each has area 1. What is $FI^2$?




**A)** $\frac{7}{3}$ **B)** $8 - 4\sqrt{2}$ **C)** $1 + \sqrt{2}$ **D)** $\frac{7}{4}\sqrt{2}$ **E)** $2\sqrt{2}$

**Q22**. What is the remainder when $2^{202} + 202$ is divided by $2^{101} + 2^{51} + 1$?

**A)** 100 **B)** 101 **C)** 200 **D)** 201 **E)** 202

**Q23**. Square $ABCD$ in the coordinate plane has vertices at the points $A(1,1), B(-1,1), C(-1,-1)$, and $D(1,-1)$. Consider the following four transformations:

- $L$, a rotation of 90° counterclockwise around the origin;
- $R$, a rotation of 90° clockwise around the origin;
- $H$, a reflection across the $x$-axis; and
- $V$, a reflection across the $y$-axis.

Each of these transformations maps the squares onto itself, but the positions of the labeled vertices will change. For example, applying $R$ and then $V$ would send the vertex $A$ at $(1, 1)$ to $(-1, -1)$ and would send the vertex $B$ at $(-1, 1)$ to itself. How many sequences of 20 transformations chosen from $\{L, R, H, V\}$ will send all of the labeled vertices back to their original positions? (For example, $R, R, V, H$ is one sequence of 4 transformations that will send the vertices back to their original positions.)

**A)** $2^{37}$ **B)** $3 \cdot 2^{36}$ **C)** $2^{38}$ **D)** $3 \cdot 2^{37}$ **E)** $2^{39}$

**Q24**. How many positive integers $n$ satisfy

$$\frac{n + 1000}{70} = \lfloor \sqrt{n} \rfloor?$$

(Recall that $\lfloor x \rfloor$ is the greatest integer not exceeding $x$).

**A)** 2 **B)** 4 **C)** 6 **D)** 30 **E)** 32

**Q25**. Let $D(n)$ denote the number of ways of writing the positive integer $n$ as a product

$$n = f_1 \cdot f_2 \cdots f_k,$$

where $k \geq 1$, the $f_i$ are integers strictly greater than 1, and the order in which the factors are listed matters (that is, two representations that differ only in the order of the factors are counted as distinct). For example, the number 6 can be written as 6, $2 \cdot 3$, and $3 \cdot 2$, so $D(6) = 3$. What is $D(96)$?

**A)** 112 **B)** 128 **C)** 144 **D)** 172 **E)** 184

**Q1**. What is the value of

$$2^{\left(0^{\left(1^9\right)}\right)}+\left(\left(2^0\right)^1\right)^9?$$

**A)** 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

**Q2**. What is the hundreds digit of $(20!-15!)$?

**A)** 0 **B)** 1 **C)** 2 **D)** 4 **E)** 5

**Q3**. Ana and Bonita were born on the same date in different years, $n$ years apart. Last year Ana was 5 times as old as Bonita. This year Ana's age is the square of Bonita's age. What is $n$?

**A)** 3 **B)** 5 **C)** 9 **D)** 12 **E)** 15

**Q4**. A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn from the box without replacement to guarantee that at least 15 balls of a single color will be drawn?

**A)** 75 **B)** 76 **C)** 79 **D)** 84 **E)** 91

**Q5**. What is the greatest number of consecutive integers whose sum is 45?

**A)** 9 **B)** 25 **C)** 45 **D)** 90 **E)** 120

**Q6**. For how many of the following types of quadrilaterals does there exist a point in the plane of the quadrilateral that is equidistant from all four vertices of the quadrilateral?

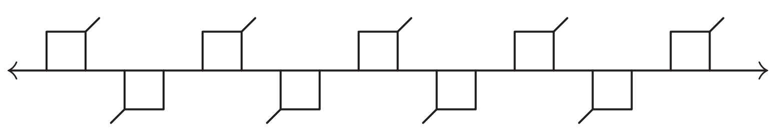
- a square
- a rectangle that is not a square
- a rhombus that is not a square
- a parallelogram that is not a rectangle or a rhombus
- an isosceles trapezoid that is not a parallelogram

**A)** 0 **B)** 2 **C)** 3 **D)** 4 **E)** 5

**Q7**. Two lines with slopes $\frac{1}{2}$ and 2 intersect at $(2,2)$. What is the area of the triangle enclosed by these two lines and the line $x+y=10$?

**A)** 4 **B)** $4\sqrt{2}$ **C)** 6 **D)** 8 **E)** $6\sqrt{2}$

**Q8**. The figure below shows line $\ell$ with a regular, infinite, recurring pattern of squares and line segments.



How many of the following four kinds of rigid motion transformations of the plane in which this figure is drawn, other than the identity transformation, will transform this figure into itself?

- some rotation around a point of line $\ell$
- some translation in the direction parallel to line $\ell$
- the reflection across line $\ell$
- some reflection across a line perpendicular to line $\ell$

**A)** 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

**Q9**. What is the greatest three-digit positive integer $n$ for which the sum of the first $n$ positive integers is _not_ a divisor of the product of the first $n$ positive integers?

**A)** 995 **B)** 996 **C)** 997 **D)** 998 **E)** 999

**Q10**. A rectangular floor that is 10 feet wide and 17 feet long is tiled with 170 one-foot square tiles. A bug walks from one corner to the opposite corner in a straight line. Including the first and the last tile, how many tiles does the bug visit?

**A)** 17 **B)** 25 **C)** 26 **D)** 27 **E)** 28

**Q11**. How many positive integer divisors of $201^9$ are perfect squares or perfect cubes (or both)?

**A)** 32 **B)** 36 **C)** 37 **D)** 39 **E)** 41

**Q12**. Melanie computes the mean $\mu$, the median $M$, and the modes of the 365 values that are the dates in the months of 2019. Thus her data consist of 12 1s, 12 2s, . . . , 12 28s, 11 29s, 11 30s, and 7 31s. Let $d$ be the median of the modes. Which of the following statements is true?

**A)** $\mu < d < M$ **B)** $M < d < \mu$ **C)** $d = M = \mu$ **D)** $d < M < \mu$ **E)** $d < \mu < M$

**Q13**. Let $\triangle ABC$ be an isosceles triangle with $BC = AC$ and $\angle ACB = 40^\circ$. Construct the circle with diameter $\overline{BC}$, and let $D$ and $E$ be the other intersection points of the circle with the sides $\overline{AC}$ and $\overline{AB}$, respectively. Let $F$ be the intersection of the diagonals of the quadrilateral $BCDE$. What is the degree measure of $\angle BFC$?

**A)** 90 **B)** 100 **C)** 105 **D)** 110 **E)** 120

**Q14**. For a set of four distinct lines in a plane, there are exactly $N$ distinct points that lie on two or more of the lines. What is the sum of all possible values of $N$?

**A)** 14 **B)** 16 **C)** 18 **D)** 19 **E)** 21

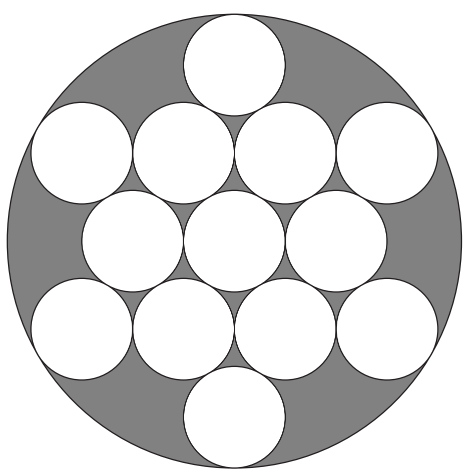
**Q15**. A sequence of numbers is defined recursively by $a_1 = 1$, $a_2 = \frac{3}{7}$, and

$$a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$$

for all $n \geq 3$ Then $a_{2019}$ can be written as $\frac{p}{q}$, where $p$ and $q$ are relatively prime positive integers. What is $p + q$?

**A)** 2020 **B)** 4039 **C)** 6057 **D)** 6061 **E)** 8078

**Q16**. The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all the circles of radius 1?



**A)** $4\pi\sqrt{3}$ **B)** $7\pi$ **C)** $\pi\left(3\sqrt{3}+2\right)$ **D)** $10\pi\left(\sqrt{3}-1\right)$ **E)** $\pi\left(\sqrt{3}+6\right)$

**Q17**. A child builds towers using identically shaped cubes of different color. How many different towers with a height 8 cubes can the child build with 2 red cubes, 3 blue cubes, and 4 green cubes? (One cube will be left out.)

**A)** 24 **B)** 288 **C)** 312 **D)** 1,260 **E)** 40,320

**Q18**. For some positive integer $k$, the repeating base-$k$ representation of the (base-ten) fraction $\frac{7}{51}$ is $0.\overline{23}_k = 0.232323\ldots k$. What is $k$?

**A)** 13 **B)** 14 **C)** 15 **D)** 16 **E)** 17

**Q19**. What is the least possible value of

$$(x+1)(x+2)(x+3)(x+4)+2019$$

where $x$ is a real number?

**A)** 2017 **B)** 2018 **C)** 2019 **D)** 2020 **E)** 2021

**Q20**. The numbers $1, 2, \ldots, 9$ are randomly placed into the 9 squares of a $3 \times 3$ grid. Each square gets one number, and each of the numbers is used once. What is the probability that the sum of the numbers in each row and each column is odd?

**A)** $\frac{1}{21}$ **B)** $\frac{1}{14}$ **C)** $\frac{5}{63}$ **D)** $\frac{2}{21}$ **E)** $\frac{1}{7}$

**Q21**. A sphere with center $O$ has radius 6. A triangle with sides of length $15, 15$, and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between $O$ and the plane determined by the triangle?

**A)** $2\sqrt{3}$ **B)** 4 **C)** $3\sqrt{2}$ **D)** $2\sqrt{5}$ **E)** 5

**Q22**. Real numbers between 0 and 1, inclusive, are chosen in the following manner. A fair coin is flipped. If it lands heads, then it is flipped again and the chosen number is 0 if the second flip is heads, and 1 if the second flip is tails. On the other hand, if the first coin flip is tails, then the number is chosen uniformly at random from the closed interval $[0, 1]$. Two random numbers $x$ and $y$ are chosen independently in this manner. What is the probability that $|x-y| > \frac{1}{2}$?

**A)** $\frac{1}{3}$ **B)** $\frac{7}{16}$ **C)** $\frac{1}{2}$ **D)** $\frac{9}{16}$ **E)** $\frac{2}{3}$

**Q23**. Travis has to babysit the terrible Thompson triplets. Knowing that they love big numbers, Travis devises a counting game for them. First Tadd will say the number 1, then Todd must say the next two numbers (2 and 3), then Tucker must say the next three numbers (4, 5, 6), then Tadd must say the next four numbers (7, 8, 9, 10), and the process continues to rotate through the three children in order, each saying one more number than the previous child did, until the number 10,000 is reached. What is the 2019th number said by Tadd?

**A)** 5743 **B)** 5885 **C)** 5979 **D)** 6001 **E)** 6011

**Q24**. Let $p$, $q$, and $r$ be the distinct roots of the polynomial $x^3 - 22x^2 + 80x - 67$. It is given that there exist real numbers $A$, $B$, and $C$ such that

$$\frac{1}{s^3 - 22s^2 + 80s - 67} = \frac{A}{s-p} + \frac{B}{s-q} + \frac{C}{s-r}$$

for all $s \notin \{p, q, r\}$. What is $\frac{1}{A} + \frac{1}{B} + \frac{1}{C}$?

**A)** 243 **B)** 244 **C)** 245 **D)** 246 **E)** 247

**Q25**. For how many integers $n$ between 1 and 50, inclusive, is

$$\frac{(n^2-1)!}{(n!)^n}$$

an integer? (Recall that $0! = 1$.)

**A)** 31 **B)** 32 **C)** 33 **D)** 34 **E)** 35

**Q1**. Alicia had two containers. The first was $\frac{5}{6}$ full of water and the second was empty. She poured all the water from the first container into the second container, at which point the second container was $\frac{3}{4}$ full of water. What is the ratio of the volume of the first container to the volume of the second container?

**A)** $\frac{5}{8}$ **B)** $\frac{4}{5}$ **C)** $\frac{7}{8}$ **D)** $\frac{9}{10}$ **E)** $\frac{11}{12}$

**Q2**. Consider the statement, "If $n$ is not prime, then $n-2$ is prime." Which of the following values of $n$ is a counterexample to this statement?

**A)** 11 **B)** 15 **C)** 19 **D)** 21 **E)** 27

**Q3**. In a high school with 500 students, 40% of the seniors play a musical instrument, while 30% of the non-seniors do not play a musical instrument. In all, 46.8% of the students do not play a musical instrument. How many non-seniors play a musical instrument?

**A)** 66 **B)** 154 **C)** 186 **D)** 220 **E)** 266

**Q4**. All lines with equation $ax+by=c$ such that $a, b, c$ form an arithmetic progression pass through a common point. What are the coordinates of that point?

**A)** $(-1,2)$ **B)** $(0,1)$ **C)** $(1,-2)$ **D)** $(1,0)$ **E)** $(1,2)$

**Q5**. Triangle $ABC$ lies in the first quadrant. Points $A$, $B$, and $C$ are reflected across the line $y=x$ to points $A'$, $B'$, and $C'$, respectively. Assume that none of the vertices of the triangle lie on the line $y=x$. Which of the following statements is **not** always true?

**A)** Triangle $A'B'C'$ lies in the first quadrant.
**B)** Triangles $ABC$ and $A'B'C'$ have the same area.
**C)** The slope of line $AA'$ is $-1$.
**D)** The slopes of lines $AA'$ and $CC'$ are the same.
**E)** Lines $AB$ and $A'B'$ are perpendicular to each other.

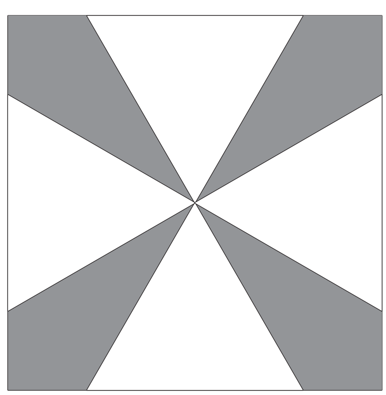
**Q6**. There is a positive integer $n$ such that $(n+1)!+(n+2)!=n!\cdot 440$. What is the sum of the digits of $n$?

**A)** 3 **B)** 8 **C)** 10 **D)** 11 **E)** 12

**Q7**. Each piece of candy in a store costs a whole number of cents. Casper has exactly enough money to buy either 12 pieces of red candy, 14 pieces of green candy, 15 pieces of blue candy, or $n$ pieces of purple candy. A piece of purple candy costs 20 cents. What is the smallest possible value of $n$?

**A)** 18 **B)** 21 **C)** 24 **D)** 25 **E)** 28

**Q8**. The figure below shows a square and four equilateral triangles, with each triangle having a side lying on a side of the square, such that each triangle has side length 2 and the third vertices of the triangles meet at the center of the square. The region inside the square but outside the triangles is shaded. What is the area of the shaded region?



**A)** 4 **B)** $12-4\sqrt{3}$ **C)** $3\sqrt{3}$ **D)** $4\sqrt{3}$ **E)** $16-4\sqrt{3}$

**Q9**. The function $f$ is defined by

$$f(x)=\lfloor |x| \rfloor - |\lfloor x \rfloor|$$

for all real numbers $x$, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number $r$. What is the range of $f$?

**A)** $\{-1,0\}$

**B)** The set of nonpositive integers

**C)** $\{-1,0,1\}$

**D)** $\{0\}$

**E)** The set of nonnegative integers

**Q10**. In a given plane, points $A$ and $B$ are 10 units apart. How many points $C$ are there in the plane such that the perimeter of $\triangle ABC$ is 50 units and the area of $\triangle ABC$ is 100 square units?

**A)** 0 **B)** 2 **C)** 4 **D)** 8 **E)** infinitely many

**Q11**. Two jars each contain the same number of marbles, and every marble is either blue or green. In Jar 1 the ratio of blue to green marbles is $9:1$, and the ratio of blue to green marbles in Jar 2 is $8:1$. There are 95 green marbles in all. How many more blue marbles are in Jar 1 than in Jar 2?

**A)** 5 **B)** 10 **C)** 25 **D)** 45 **E)** 50

**Q12**. What is the greatest possible sum of the digits in the base-seven representation of a positive integer less than 2019?

**A)** 11 **B)** 14 **C)** 22 **D)** 23 **E)** 27

**Q13**. What is the sum of all real numbers $x$ for which the median of the numbers $4, 6, 8, 17$, and $x$ is equal to the mean of those five numbers?

**A)** $-5$ **B)** 0 **C)** 5 **D)** $\frac{15}{4}$ **E)** $\frac{35}{4}$

**Q14**. The base-ten representation for 19! is $121,6T5,100,40M,832,H00$, where $T$, $M$, and $H$ denote digits that are not given. What is $T+M+H$?

**A)** 3 **B)** 8 **C)** 12 **D)** 14 **E)** 17

**Q15**. Right triangles $T_1$ and $T_2$, have areas of 1 and 2, respectively. A side of $T_1$ is congruent to a side of $T_2$, and a different side of $T_1$ is congruent to a different side of $T_2$. What is the square of the product of the lengths of the other (third) side of $T_1$ and $T_2$?

**A)** $\frac{28}{3}$ **B)** 10 **C)** $\frac{32}{3}$ **D)** $\frac{34}{3}$ **E)** 12

**Q16**. In $\triangle ABC$ with a right angle at $C$, point $D$ lies in the interior of $\overline{AB}$ and point $E$ lies in the interior of $\overline{BC}$ so that $AC=CD$, $DE=EB$, and the ratio $AC:DE=4:3$. What is the ratio $AD:DB$?

**A)** $2:3$ **B)** $2:\sqrt{5}$ **C)** $1:1$ **D)** $3:\sqrt{5}$ **E)** $3:2$

**Q17**. A red ball and a green ball are randomly and independently tossed into bins numbered with the positive integers so that for each ball, the probability that it is tossed into bin $k$ is $2^{-k}$ for $k=1,2,3....$ What is the probability that the red ball is tossed into a higher-numbered bin than the green ball?

**A)** $\frac{1}{4}$ **B)** $\frac{2}{7}$ **C)** $\frac{1}{3}$ **D)** $\frac{3}{8}$ **E)** $\frac{3}{7}$

**Q18**. Henry decides one morning to do a workout, and he walks $\frac{3}{4}$ of the way from his home to his gym. The gym is 2 kilometers away from Henry's home. At that point, he changes his mind and walks $\frac{3}{4}$ of the way from where he is back toward home. When he reaches that point, he changes his mind again and walks $\frac{3}{4}$ of the distance from there back toward the gym. If Henry keeps changing his mind when he has walked $\frac{3}{4}$ of the distance toward either the gym or home from the point where he last changed his mind, he will get very close to walking back and forth between a point $A$ kilometers from home and a point $B$ kilometers from home. What is $|A-B|$?

**A)** $\frac{2}{3}$ **B)** 1 **C)** $1\frac{1}{5}$ **D)** $1\frac{1}{4}$ **E)** $1\frac{1}{2}$

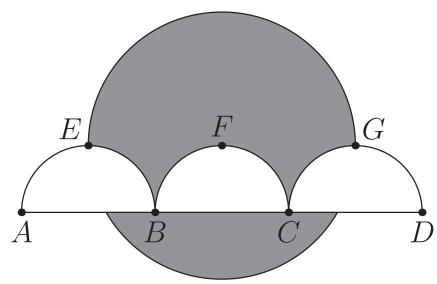
**Q19**. Let $S$ be the set of all positive integer divisors of $100,000$. How many numbers are the product of two distinct elements of $S$?

**A)** 98 **B)** 100 **C)** 117 **D)** 119 **E)** 121

**Q20**. As shown in the figure, line segment $\overline{AD}$ is trisected by points $B$ and $C$ so that $AB = BC = CD = 2$. Three semicircles of radius 1, $\widehat{AEB}$, $\widehat{BFC}$, and $\widehat{CGD}$, have their diameters on $\overline{AD}$, and are tangent to line $EG$ at $E, F$, and $G$, respectively. A circle of radius 2 has its center on $F$. The area of the region inside the circle but outside the three semicircles, shaded in the figure, can be expressed in the form

$$\frac{a}{b} \cdot \pi - \sqrt{c} + d,$$

where $a, b, c$, and $d$ are positive integers and $a$ and $b$ are relatively prime. What is $a+b+c+d$?




**A)** 13 **B)** 14 **C)** 15 **D)** 16 **E)** 17

**Q21**. Debra flips a fair coin repeatedly, keeping track of how many heads and how many tails she has seen in total, until she gets either two heads in a row or two tails in a row, at which point she stops flipping. What is the probability that she gets two heads in a row but she sees a second tail before she sees a second head?

**A)** $\frac{1}{36}$ **B)** $\frac{1}{24}$ **C)** $\frac{1}{18}$ **D)** $\frac{1}{12}$ **E)** $\frac{1}{6}$

**Q22**. Raashan, Sylvia, and Ted play the following game. Each starts with \$1. A bell rings every 15 seconds, at which time each of the players who currently have money simultaneously chooses one of the other two players independently and at random and gives \$1 to that player. What is the probability that after the bell has rung 2019 times, each player will have \$1? (For example, Raashan and Ted may each decide to give \$1 to Sylvia, and Sylvia may decide to give her dollar to Ted, at which point Raashan will have \$0, Sylvia will have \$2, and Ted will have \$1, and that is the end of the first round of play. In the second round Rashaan has no money to give, but Sylvia and Ted might choose each other to give their \$1 to, and the holdings will be the same at the end of the second round.)

**A)** $\frac{1}{7}$ **B)** $\frac{1}{4}$ **C)** $\frac{1}{3}$ **D)** $\frac{1}{2}$ **E)** $\frac{2}{3}$

**Q23**. Points $A(6,13)$ and $B(12,11)$ lie on circle $\omega$ in the plane. Suppose that the tangent lines to $\omega$ at $A$ and $B$ intersect at a point on the $x$-axis. What is the area of $\omega$?

**A)** $\frac{83\pi}{8}$ **B)** $\frac{21\pi}{2}$ **C)** $\frac{85\pi}{8}$ **D)** $\frac{43\pi}{4}$ **E)** $\frac{87\pi}{8}$

**Q24**. Define a sequence recursively by $x_0 = 5$ and

$$x_{n+1} = \frac{x_n^2 + 5x_n + 4}{x_n + 6}$$

for all nonnegative integers $n$. Let $m$ be the least positive integer such that

$$x_m \leq 4 + \frac{1}{2^{20}}.$$

In which of the following intervals does $m$ lie?

**A)** $[9, 26]$ **B)** $[27, 80]$ **C)** $[81, 242]$ **D)** $[243, 728]$ **E)** $[729, \infty)$

**Q25**. How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

**A)** 55 **B)** 60 **C)** 65 **D)** 70 **E)** 75

**Q1**. What is the value of

$$\left(\left((2+1)^{-1}+1\right)^{-1}+1\right)^{-1}+1?$$

**A)** $\frac{5}{8}$ **B)** $\frac{11}{7}$ **C)** $\frac{8}{5}$ **D)** $\frac{18}{11}$ **E)** $\frac{15}{8}$

**Q2**. Liliane has 50% more soda than Jacqueline, and Alice has 25% more soda than Jacqueline. What is the relationship between the amounts of soda that Liliane and Alica have?

**A)** Liliane has 20% more soda than Alice.

**B)** Liliane has 25% more soda than Alice.

**C)** Liliane has 45% more soda than Alice.

**D)** Liliane has 75% more soda than Alice.

**E)** Liliane has 100% more soda than Alice.

**Q3**. A unit of blood expires after $10! = 10 \cdot 9 \cdot 8 \cdots 1$ seconds. Yasin donates a unit of blood at noon of January 1. On what day does his unit of blood expire?

**A)** January 2 **B)** January 12 **C)** January 22 **D)** February 11 **E)** February 12

**Q4**. How many ways can a student schedule 3 mathematics courses – algebra, geometry, and number theory – in a 6-period day if no two mathematics courses can be taken in consecutive periods? (What courses the student takes during the other 3 periods is of no concern here.)

**A)** 3 **B)** 6 **C)** 12 **D)** 18 **E)** 24

**Q5**. Alice, Bob, and Charlie were on a hike and were wondering how far away the nearest town was. When Alice said, "We are at least 6 miles away," Bob replied, "We are at most 5 miles away." Charlie then remarked, "Actually the nearest town is at most 4 miles away." It turned out that none of the three statements were true. Let $d$ be the distance in miles to the nearest town. Which of the following intervals is the set of all possible values of $d$?

**A)** $(0,4)$ **B)** $(4,5)$ **C)** $(4,6)$ **D)** $(5,6)$ **E)** $(5,\infty)$

**Q6**. Sangho uploaded a video to a website where viewers can vote that they like or dislike a video. Each video begins with a score of 0, and the score increases by 1 for each like vote and decreases by 1 for each dislike vote. At one point Sangho saw that his video had a score of 90, and that 65% of the votes cast on his video were like votes. How many votes had been cast on Sangho's video at that point?

**A)** 200 **B)** 300 **C)** 400 **D)** 500 **E)** 600

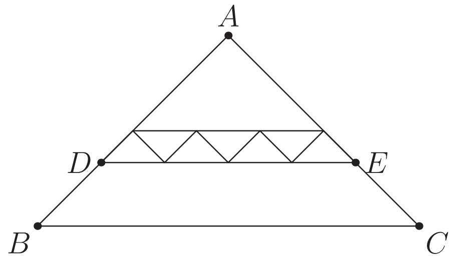
**Q7**. For how many (not necessarily positive) integer values of $n$ is the value of $4000 \cdot \left(\frac{2}{5}\right)^n$ an integer?

**A)** 3 **B)** 4 **C)** 6 **D)** 8 **E)** 9

**Q8**. Joe has a collection of 23 coins, consisting of 5-cent coins, 10-cent coins, and 25-cent coins. He has 3 more 10-cent coins than 5-cent coins, and the total value of his collection is 320 cents. How many more 25-cent coins does Joe have than 5-cent coins?

**A)** 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

**Q9**. All of the triangles in the diagram below are similar to isosceles triangle $ABC$, in which $AB = AC$. Each of the 7 smallest triangles has area 1, and $\triangle ABC$ has area 40. What is the area of trapezoid $DBCE$?




**A)** 16 **B)** 18 **C)** 20 **D)** 22 **E)** 24

**Q10**. Suppose that real number $x$ satisfies

$$\sqrt{49-x^2}-\sqrt{25-x^2}=3$$

What is the value of $\sqrt{49-x^2}+\sqrt{25-x^2}$?

**A)** 8 **B)** $\sqrt{33}+8$ **C)** 9 **D)** $2\sqrt{10}+4$ **E)** 12

**Q11**. When 7 fair standard 6-sided dice are thrown, the probability that the sum of the numbers on the top faces is 10 can be written as

$$\frac{n}{6^7},$$

where $n$ is a positive integer. What is $n$?

**A)** 42 **B)** 49 **C)** 56 **D)** 63 **E)** 84

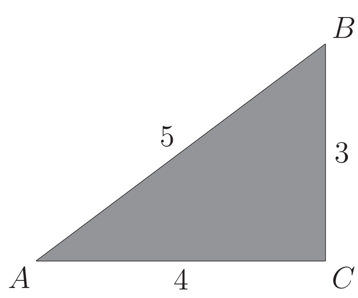
**Q12**. How many ordered pairs of real numbers $(x, y)$ satisfy the following system of equations?

$$x+3y=3$$

$$||x|-|y||=1$$

**A)** 1 **B)** 2 **C)** 3 **D)** 4 **E)** 8

**Q13**. A paper triangle with sides of lengths 3, 4, and 5 inches, as shown, is folded so that point $A$ falls on point $B$. What is the length in inches of the crease?



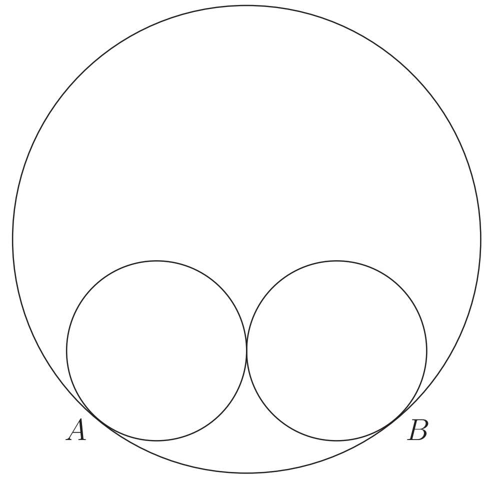

**A)** $1+\frac{1}{2}\sqrt{2}$ **B)** $\sqrt{3}$ **C)** $\frac{7}{4}$ **D)** $\frac{15}{8}$ **E)** 2

**Q14**. What is the greatest integer less than or equal to

$$\frac{3^{100}+2^{100}}{3^{96}+2^{96}}?$$

**A)** 80 **B)** 81 **C)** 96 **D)** 97 **E)** 625

**Q15**. Two circles of radius 5 are externally tangent to each other and are internally tangent to a circle of radius 13 at points $A$ and $B$, as shown in the diagram. The distance $AB$ can be written in the form $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. What is $m+n$?




**A)** 21 **B)** 29 **C)** 58 **D)** 69 **E)** 93

**Q16**. Right triangle $ABC$ has leg lengths $AB = 20$ and $BC = 21$. Including $\overline{AB}$ and $\overline{BC}$, how many line segments with integer length can be drawn from vertex $B$ to a point on hypotenuse $\overline{AC}$?

**A)** 5 **B)** 8 **C)** 12 **D)** 13 **E)** 15

**Q17**. Let $S$ be a set of 6 integers taken from $\{1, 2, \ldots, 12\}$ with the property that if $a$ and $b$ are elements of $S$ with $a < b$, then $b$ is not a multiple of $a$. What is the least possible value of an element in $S$?

**A)** 2 **B)** 3 **C)** 4 **D)** 5 **E)** 7

**Q18**. How many nonnegative integers can be written in the form

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5 + a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2 + a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

where $a_i \in \{-1, 0, 1\}$ for $0 \le i \le 7$?

**A)** 512 **B)** 729 **C)** 1094 **D)** 3281 **E)** 59,048

**Q19**. A number $m$ is randomly selected from the set $\{11, 13, 15, 17, 19\}$, and a number $n$ is randomly selected from $\{1999, 2000, 2001, \ldots, 2018\}$. What is the probability that $m^n$ has a units digit of 1?

**A)** $\frac{1}{5}$ **B)** $\frac{1}{4}$ **C)** $\frac{3}{10}$ **D)** $\frac{7}{20}$ **E)** $\frac{2}{5}$

**Q20**. A scanning code consists of a $7 \times 7$ grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of 49 squares. A scanning code is called *symmetric* if its look does not change when the entire square is rotated by a multiple of $90^\circ$ counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides. What is the total number of possible symmetric scanning codes?

**A)** 510 **B)** 1022 **C)** 8190 **D)** 8192 **E)** 65,534

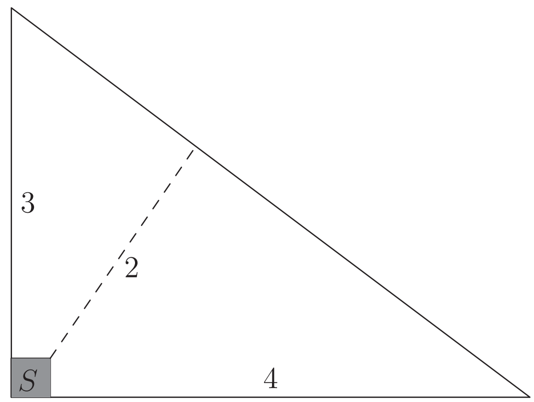
**Q21**. Which of the following describes the set of values of $a$ for which the curves $x^2 + y^2 = a^2$ and $y = x^2 - a$ in the real $xy$-plane intersect at exactly 3 points?

**A)** $a = \frac{1}{4}$ **B)** $\frac{1}{4} < a < \frac{1}{2}$ **C)** $a > \frac{1}{4}$ **D)** $a = \frac{1}{2}$ **E)** $a > \frac{1}{2}$

**Q22**. Let $a, b, c$, and $d$ be positive integers such that $\gcd(a, b) = 24$, $\gcd(b, c) = 36$, $\gcd(c, d) = 54$, and $70 < \gcd(d, a) < 100$. Which of the following must be a divisor of $a$?

**A)** 5 **B)** 7 **C)** 11 **D)** 13 **E)** 17

**Q23**. Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square $S$ so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from $S$ to the hypotenuse is 2 units. What fraction of the field is planted?




**A)** $\frac{25}{27}$ **B)** $\frac{26}{27}$ **C)** $\frac{73}{75}$ **D)** $\frac{145}{147}$ **E)** $\frac{74}{75}$

**Q24**. Triangle $ABC$ with $AB = 50$ and $AC = 10$ has area 120. Let $D$ be the midpoint of $\overline{AB}$, and let $E$ be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at $F$ and $G$, respectively. What is the area of quadrilateral $FDBG$?

**A)** 60 **B)** 65 **C)** 70 **D)** 75 **E)** 80

**Q25**. For a positive integer $n$ and nonzero digits $a$, $b$, and $c$, let $A_n$ be the $n$-digit integer each of whose digits is equal to $a$; let $B_n$ be the $n$-digit integer each of whose digits is equal to $b$, and let $C_n$ be the $2n$-digit (not $n$-digit) integer each of whose digits is equal to $c$. What is the greatest possible value of $a + b + c$ for which there are at least two values of $n$ such that $C_n - B_n = A_n^2$?

**A)** 12 **B)** 14 **C)** 16 **D)** 18 **E)** 20

**Q1**. Kate bakes a 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?

**A)** 90 **B)** 100 **C)** 180 **D)** 200 **E)** 360

**Q2**. Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph (miles per hour), and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes?

**A)** 64 **B)** 65 **C)** 66 **D)** 67 **E)** 68

**Q3**. In the expression $(\_\_\_\_ \times \_\_\_\_) + (\_\_\_\_ \times \_\_\_\_)$ each blank is to be filled in with one of the digits $1, 2, 3,$ or $4$, with each digit being used once. How many different values can be obtained?

**A)** 2 **B)** 3 **C)** 4 **D)** 6 **E)** 24

**Q4**. A three-dimensional rectangular box with dimensions $X$, $Y$, and $Z$ has faces whose surface areas are 24, 24, 48, 48, 72, and 72 square units. What is $X + Y + Z$?

**A)** 18 **B)** 22 **C)** 24 **D)** 30 **E)** 36

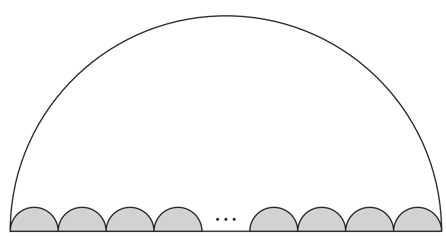
**Q5**. How many subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ contain at least one prime number?

**A)** 128 **B)** 192 **C)** 224 **D)** 240 **E)** 256

**Q6**. A box contains 5 chips, numbered 1, 2, 3, 4, and 5. Chips are drawn randomly one at a time without replacement until the sum of the values drawn exceeds 4. What is the probability that 3 draws are required?

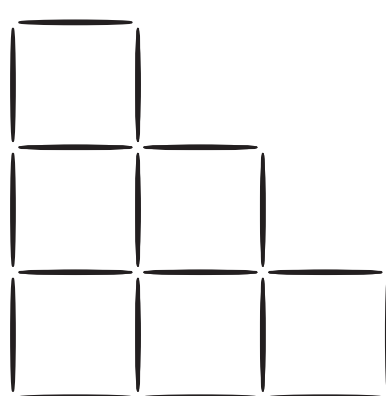
**A)** $\frac{1}{15}$ **B)** $\frac{1}{10}$ **C)** $\frac{1}{6}$ **D)** $\frac{1}{5}$ **E)** $\frac{1}{4}$

**Q7**. In the figure below, $N$ congruent semicircles lie on the diameter of a large semicircle, with their diameters covering the diameter of the large semicircle with no overlap. Let $A$ be the combined area of the small semicircles and $B$ be the area of the region inside the large semicircle but outside the semicircles. The ratio $A : B$ is $1 : 18$. What is $N$?



**A)** 16 **B)** 17 **C)** 18 **D)** 19 **E)** 36

**Q8**. Sara makes a staircase out of toothpicks as shown:



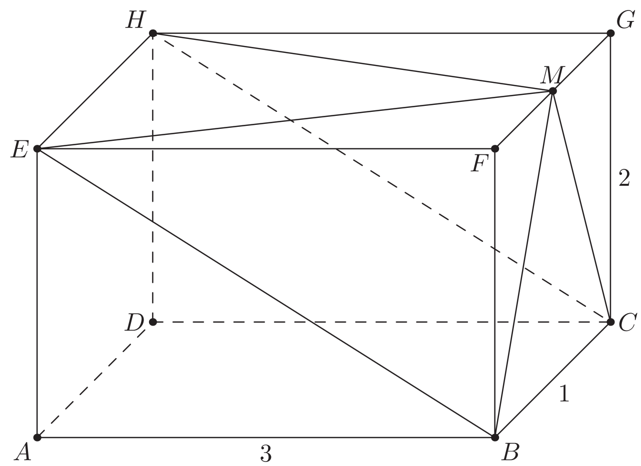
This is a 3-step staircase and uses 18 toothpicks. How many steps would be in a staircase that used 180 toothpicks?

**A)** 10 **B)** 11 **C)** 12 **D)** 24 **E)** 30

**Q9**. The faces of each of 7 standard dice are labeled with the integers from 1 to 6. Let $p$ be the probability that when all 7 dice are rolled, the sum of the numbers on the top faces is 10. What other sum occurs with the same probability $p$?

**A)** 13 **B)** 26 **C)** 32 **D)** 39 **E)** 42

**Q10**. In the rectangular parallelepiped shown, $AB = 3$, $BC = 1$, and $CG = 2$. Point $M$ is the midpoint of $\overline{FG}$. What is the volume of the rectangular pyramid with base $BCHE$ and apex $M$?




**A)** 1 **B)** $\frac{4}{3}$ **C)** $\frac{3}{2}$ **D)** $\frac{5}{3}$ **E)** 2

**Q11**. Which of the following expressions is never a prime number when $p$ is a prime number?

**A)** $p^2 + 16$ **B)** $p^2 + 24$ **C)** $p^2 + 26$ **D)** $p^2 + 46$ **E)** $p^2 + 96$

**Q12**. Line segment $\overline{AB}$ is a diameter of a circle with $AB = 24$. Point $C$, not equal to $A$ or $B$, lies on the circle. As point $C$ moves around the circle, the centroid (center of mass) of $\triangle ABC$ traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?

**A)** 25 **B)** 38 **C)** 50 **D)** 63 **E)** 75

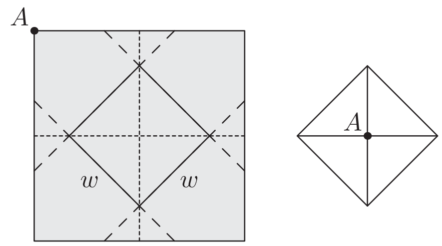
**Q13**. How many of the first 2018 numbers in the sequence $101, 1001, 10001, 100001, \ldots$ are divisible by 101?

**A)** 253 **B)** 504 **C)** 505 **D)** 506 **E)** 1009

**Q14**. A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

**A)** 202 **B)** 223 **C)** 224 **D)** 225 **E)** 234

**Q15**. A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point $A$ in the figure on the right. The box has base length $w$ and height $h$. What is the area of the sheet of wrapping paper?




**A)** $2(w+h)^2$ **B)** $\frac{(w+h)^2}{2}$ **C)** $2w^2+4wh$ **D)** $2w^2$ **E)** $w^2h$

**Q16**. Let $a_1, a_2, \dots, a_{2018}$ be a strictly increasing sequence of positive integers such that

$$a_1 + a_2 + \cdots + a_{2018} = 2018^{2018}.$$

What is the remainder when $a_1^3 + a_2^3 + \cdots + a_{2018}^3$ is divided by 6?

**A)** 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

**Q17**. In rectangle $PQRS$, $PQ = 8$ and $QR = 6$. Points $A$ and $B$ lie on $\overline{PQ}$, points $C$ and $D$ lie on $\overline{QR}$, points $E$ and $F$ lie on $\overline{RS}$, and points $G$ and $H$ lie on $\overline{SP}$ so that $AP = BQ < 4$ and the convex octagon $ABCDEFGH$ is equilateral. The length of a side of this octagon can be expressed in the form $k + m\sqrt{n}$, where $k$, $m$, and $n$ are integers and $n$ is not divisible by the square of any prime. What is $k + m + n$?

**A)** 1 **B)** 7 **C)** 21 **D)** 92 **E)** 106

**Q18**. Three young brother-sister pairs from different families need to take a trip in a van. These six children will occupy the second and third rows in the van, each of which has three seats. To avoid disruptions, siblings may not sit right next to each other in the same row, and no child may sit directly in front of his or her sibling. How many seating arrangements are possible for this trip?

**A)** 60 **B)** 72 **C)** 92 **D)** 96 **E)** 120

**Q19**. Joey and Chloe and their daughter Zoe all have the same birthday. Joey is 1 year older than Chloe, and Zoe is exactly 1 year old today. Today is the first of the 9 birthdays on which Chloe's age will be an integral multiple of Zoe's age. What will be the sum of the two digits of Joey's age the next time his age is a multiple of Zoe's age?

**A)** 7 **B)** 8 **C)** 9 **D)** 10 **E)** 11

**Q20**. A function $f$ is defined recursively by $f(1) = f(2) = 1$ and

$$f(n) = f(n-1) - f(n-2) + n$$

for all integers $n \geq 3$. What is $f(2018)$?

**A)** 2016 **B)** 2017 **C)** 2018 **D)** 2019 **E)** 2020

**Q21**. Mary chose an even 4-digit number $n$. She wrote down all the divisors of $n$ in increasing order from left to right: $1, 2, ..., \frac{n}{2}, n$. At some moment Mary wrote 323 as a divisor of $n$. What is the smallest possible value of the next divisor written to the right of 323?

**A)** 324 **B)** 330 **C)** 340 **D)** 361 **E)** 646

**Q22**. Real numbers $x$ and $y$ are chosen independently and uniformly at random from the interval $[0, 1]$. Which of the following numbers is closest to the probability that $x, y$, and 1 are the side lengths of an obtuse triangle?

**A)** 0.21 **B)** 0.25 **C)** 0.29 **D)** 0.50 **E)** 0.79

**Q23**. How many ordered pairs $(a, b)$ of positive integers satisfy the equation

$$a \cdot b + 63 = 20 \cdot \operatorname{lcm}(a, b) + 12 \cdot \gcd(a, b),$$

where $\gcd(a, b)$ denotes the greatest common divisor of $a$ and $b$, and $\operatorname{lcm}(a, b)$ denotes their least common multiple?

**A)** 0 **B)** 2 **C)** 4 **D)** 6 **E)** 8

**Q24**. Let $ABCDEF$ be a regular hexagon with side length 1. Denote by $X$, $Y$, and $Z$ the midpoints of sides $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of $\triangle ACE$ and $\triangle XYZ$?

**A)** $\frac{3}{8}\sqrt{3}$ **B)** $\frac{7}{16}\sqrt{3}$ **C)** $\frac{15}{32}\sqrt{3}$ **D)** $\frac{1}{2}\sqrt{3}$ **E)** $\frac{9}{16}\sqrt{3}$

**Q25**. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to $x$. How many real numbers $x$ satisfy the equation $x^2 + 10,000\lfloor x \rfloor = 10,000x$?

**A)** 197 **B)** 198 **C)** 199 **D)** 200 **E)** 201

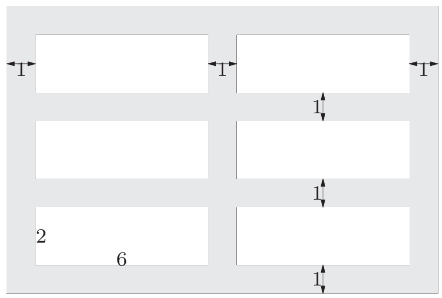
**Q1**. What is the value of $(2(2(2(2(2(2+1)+1)+1)+1)+1)+1)$?

**A)** 70 **B)** 97 **C)** 127 **D)** 159 **E)** 729

**Q2**. Pablo buys popsicles for his friends. The store sells single popsicles for \$1 each, 3-popsicle boxes for \$2 each, and 5-popsicle boxes for \$3. What is the greatest number of popsicles that Pablo can buy with \$8?

**A)** 8 **B)** 11 **C)** 12 **D)** 13 **E)** 15

**Q3**. Tamara has three rows of two 6-feet by 2-feet flower beds in her garden. The beds are separated and also surrounded by 1-foot-wide walkways, as shown on the diagram. What is the total area of the walkways, in square feet?




**A)** 72 **B)** 78 **C)** 90 **D)** 120 **E)** 150

**Q4**. Mia is "helping" her mom pick up 30 toys that are strewn on the floor. Mia's mom manages to put 3 toys into the toy box every 30 seconds, but each time immediately after those 30 seconds have elapsed, Mia takes 2 toys out of the box. How much time, in minutes, will it take Mia and her mom to put all 30 toys into the box for the first time?

**A)** 13.5 **B)** 14 **C)** 14.5 **D)** 15 **E)** 15.5

**Q5**. The sum of two nonzero real numbers is 4 times their product. What is the sum of the reciprocals of the two numbers?

**A)** 1 **B)** 2 **C)** 4 **D)** 8 **E)** 12

**Q6**. Ms. Carroll promised that anyone who got all the multiple choice questions right on the upcoming exam would receive an A on the exam. Which one of these statements necessarily follows logically?

**A)** If Lewis did not receive an A, then he got all of the multiple choice questions wrong.

**B)** If Lewis did not receive an A, then he got at least one of the multiple choice questions wrong.

**C)** If Lewis got at least one of the multiple choice questions wrong, then he did not receive an A.

**D)** If Lewis received an A, then he got all of the multiple choice questions right.

**E)** If Lewis received an A, then he got at least one of the multiple choice questions right.

**Q7**. Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

**A)** 30% **B)** 40% **C)** 50% **D)** 60% **E)** 70%

**Q8**. At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur within the group?

**A)** 240 **B)** 245 **C)** 290 **D)** 480 **E)** 490

**Q9**. Minnie rides on a flat road at 20 kilometers per hour (kph), downhill at 30 kph, and uphill at 5 kph. Penny rides on a flat road at 30 kph, downhill at 40 kph, and uphill at 10 kph. Minnie goes from town $A$ to town $B$, a distance of 10 km all uphill, then from town $B$ to town $C$, a distance of 15 km all downhill, and then back to town $A$, a distance of 20 km on the flat. Penny goes the other way around using the same route. How many more minutes does it take Minnie to complete the 45-km ride than it takes Penny?

**A)** 45 **B)** 60 **C)** 65 **D)** 90 **E)** 95

**Q10**. Joy has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15 cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?

**A)** 16 **B)** 17 **C)** 18 **D)** 19 **E)** 20

**Q11**. The region consisting of all points in three-dimensional space within 3 units of line segment $\overline{AB}$ has volume $216\pi$. What is the length $AB$?

**A)** 6 **B)** 12 **C)** 18 **D)** 20 **E)** 24

**Q12**. Let $S$ be a set of points $(x, y)$ in the coordinate plane such that two of the three quantities 3, $x+2$, and $y-4$ are equal and the third of the three quantities is no greater than this common value. Which of the following is a correct description for $S$?

**A)** a single point

**B)** two intersecting lines

**C)** three lines whose pairwise intersections are three distinct points

**D)** a triangle

**E)** three rays with a common endpoint

**Q13**. Define a sequence recursively by $F_0 = 0$, $F_1 = 1$, and $F_n =$ the remainder when $F_{n-1} + F_{n-2}$ is divided by 3, for all $n \geq 2$. Thus the sequence starts $0, 1, 1, 2, 0, 2, \ldots$ What is $F_{2017} + F_{2018} + F_{2019} + F_{2020} + F_{2021} + F_{2022} + F_{2023} + F_{2024}$?

**A)** 6 **B)** 7 **C)** 8 **D)** 9 **E)** 10

**Q14**. Every week Roger pays for a movie ticket and a soda out of his allowance. Last week, Roger's allowance was $A$ dollars. The cost of his movie ticket was 20% of the difference between $A$ and the cost of his soda, while the cost of his soda was 5% of the difference between $A$ and the cost of his movie ticket. To the nearest whole percent, what fraction of $A$ did Roger pay for his movie ticket and soda?

**A)** 9% **B)** 19% **C)** 22% **D)** 23% **E)** 25%

**Q15**. Chloe chooses a real number uniformly at random from the interval $[0, 2017]$. Independently, Laurent chooses a real number uniformly at random from the interval $[0, 4034]$. What is the probability that Laurent's number is greater than Chloe's number?

**A)** $\frac{1}{2}$ **B)** $\frac{2}{3}$ **C)** $\frac{3}{4}$ **D)** $\frac{5}{6}$ **E)** $\frac{7}{8}$

**Q16**. There are 10 horses, named Horse 1, Horse 2, . . . , Horse 10. They get their names from how many minutes it takes them to run one lap around a circular race track: Horse $k$ runs one lap in exactly $k$ minutes. At time 0 all the horses are together at the starting point on the track. The horses start running in the same direction, and they keep running around the circular track at their constant speeds. The least time $S > 0$, in minutes, at which all 10 horses will again simultaneously be at the starting point is $S = 2520$. Let $T > 0$ be the least time, in minutes, such that at least 5 of the horses are again at the starting point. What is the sum of the digits of $T$?

**A)** 2 **B)** 3 **C)** 4 **D)** 5 **E)** 6

**Q17**. Distinct points $P$, $Q$, $R$, $S$ lie on the circle $x^2 + y^2 = 25$ and have integer coordinates. The distances $PQ$ and $RS$ are irrational numbers. What is the greatest possible value of the ratio $\frac{PQ}{RS}$?

**A)** 3 **B)** 5 **C)** $3\sqrt{5}$ **D)** 7 **E)** $5\sqrt{2}$

**Q18**. Amelia has a coin that lands heads with probability $\frac{1}{3}$, and Blaine has a coin that lands on heads with probability $\frac{2}{5}$. Amelia and Blaine alternately toss their coins until someone gets a head; the first one to get a head wins. All coin tosses are independent. Amelia goes first. The probability that Amelia wins is $\frac{p}{q}$, where $p$ and $q$ are relatively prime positive integers. What is $q - p$?

**A)** 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

**Q19**. Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?

**A)** 12 **B)** 16 **C)** 28 **D)** 32 **E)** 40

**Q20**. Let $S(n)$ equal the sum of the digits of positive integer $n$. For example, $S(1507) = 13$. For a particular positive integer $n$, $S(n) = 1274$. Which of the following could be the value of $S(n + 1)$?

**A)** 1 **B)** 3 **C)** 12 **D)** 1239 **E)** 1265

**Q21**. A square with side length $x$ is inscribed in a right triangle with sides of length 3, 4, and 5 so that one vertex of the square coincides with the right-angle vertex of the triangle. A square with side length $y$ is inscribed in another right triangle with sides of length 3, 4, and 5 so that one side of the square lies on the hypotenuse of the triangle. What is $\frac{x}{y}$?

**A)** $\frac{12}{13}$ **B)** $\frac{35}{37}$ **C)** 1 **D)** $\frac{37}{35}$ **E)** $\frac{13}{12}$

**Q22**. Sides $\overline{AB}$ and $\overline{AC}$ of equilateral triangle $ABC$ are tangent to a circle at points $B$ and $C$ respectively. What fraction of the area of $\triangle ABC$ lies outside the circle?

**A)** $\frac{4\sqrt{3}\pi}{27} - \frac{1}{3}$ **B)** $\frac{\sqrt{3}}{2} - \frac{\pi}{8}$ **C)** $\frac{1}{2}$ **D)** $\sqrt{3} - \frac{2\sqrt{3}\pi}{9}$ **E)** $\frac{4}{3} - \frac{4\sqrt{3}\pi}{27}$

**Q23**. How many triangles with positive area have all their vertices at points $(i, j)$ in the coordinate plane, where $i$ and $j$ are integers between 1 and 5, inclusive?

**A)** 2128 **B)** 2148 **C)** 2160 **D)** 2200 **E)** 2300

**Q24**. For certain real numbers $a$, $b$, and $c$, the polynomial

$$g(x) = x^3 + ax^2 + x + 10$$

has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

$$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

What is $f(1)$?

**A)** $-9009$ **B)** $-8008$ **C)** $-7007$ **D)** $-6006$ **E)** $-5005$

**Q25**. How many integers between 100 and 999, inclusive, have the property that some permutation of its digits is a multiple of 11 between 100 and 999? For example, both 121 and 211 have this property.

**A)** 226 **B)** 243 **C)** 270 **D)** 469 **E)** 486

**Q1**. Mary thought of a positive two-digit number. She multiplied it by 3 and added 11. Then she switched the digits of the result, obtaining a number between 71 and 75, inclusive. What was Mary's number?

**A)** 11 **B)** 12 **C)** 13 **D)** 14 **E)** 15

**Q2**. Sofia ran 5 laps around the 400-meter track at her school. For each lap, she ran the first 100 meters at an average speed of 4 meters per second and the remaining 300 meters at an average speed of 5 meters per second. How much time did Sofia take running the 5 laps?

**A)** 5 minutes and 35 seconds

**B)** 6 minutes and 40 seconds

**C)** 7 minutes and 5 seconds

**D)** 7 minutes and 25 seconds

**E)** 8 minutes and 10 seconds

**Q3**. Real numbers $x$, $y$, and $z$ satisfy the inequalities $0 < x < 1$, $-1 < y < 0$, and $1 < z < 2$. Which of the following numbers is necessarily positive?

**A)** $y + x^2$ **B)** $y + xz$ **C)** $y + y^2$ **D)** $y + 2y^2$ **E)** $y + z$

**Q4**. Suppose that $x$ and $y$ are nonzero real numbers such that $\dfrac{3x+y}{x-3y} = -2$. What is the value of $\dfrac{x+3y}{3x-y}$?

**A)** $-3$ **B)** $-1$ **C)** 1 **D)** 2 **E)** 3

**Q5**. Camilla had twice as many blueberry jelly beans as cherry jelly beans. After eating 10 pieces of each kind, she now has three times as many blueberry jelly beans as cherry jelly beans. How many blueberry jelly beans did she originally have?

**A)** 10 **B)** 20 **C)** 30 **D)** 40 **E)** 50

**Q6**. What is the largest number of solid 2-in $\times$ 2-in $\times$ 1-in blocks that can fit in a 3-in $\times$ 2-in $\times$ 3-in box?

**A)** 3 **B)** 4 **C)** 5 **D)** 6 **E)** 7

**Q7**. Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

**A)** 2.0 **B)** 2.2 **C)** 2.8 **D)** 3.4 **E)** 4.4

**Q8**. Points $A(11, 9)$ and $B(2, -3)$ are vertices of $\triangle ABC$ with $AB = AC$. The altitude from $A$ meets the opposite side at $D(-1, 3)$. What are the coordinates of point $C$?

**A)** $(-8, 9)$ **B)** $(-4, 8)$ **C)** $(-4, 9)$ **D)** $(-2, 3)$ **E)** $(-1, 0)$

**Q9**. A radio program has a quiz consisting of 3 multiple-choice questions, each with 3 choices. A contestant wins if he or she gets 2 or more of the questions right. The contestant answers randomly to each question. What is the probability of winning?

**A)** $\dfrac{1}{27}$ **B)** $\dfrac{1}{9}$ **C)** $\dfrac{2}{9}$ **D)** $\dfrac{7}{27}$ **E)** $\dfrac{1}{2}$

**Q10**. The lines with equations $ax - 2y = c$ and $2x + by = -c$ are perpendicular and intersect at $(1, -5)$. What is $c$?

**A)** $-13$ **B)** $-8$ **C)** 2 **D)** 8 **E)** 13

**Q11**. At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

**A)** 10% **B)** 12% **C)** 20% **D)** 25% **E)** $33\frac{1}{3}\%$

**Q12**. Elmer's new car gives 50% percent better fuel efficiency, measured in kilometers per liter, than his old car. However, his new car uses diesel fuel, which is 20% more expensive per liter than the gasoline his old car used. By what percent will Elmer save money if he uses his new car instead of his old car for a long trip?

**A)** 20% **B)** $26\frac{2}{3}\%$ **C)** $27\frac{7}{9}\%$ **D)** $33\frac{1}{3}\%$ **E)** $66\frac{2}{3}\%$

**Q13**. There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes?

**A)** 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

**Q14**. An integer $N$ is selected at random in the range $1 \leq N \leq 2020$ . What is the probability that the remainder when $N^{16}$ is divided by 5 is 1?

**A)** $\frac{1}{5}$ **B)** $\frac{2}{5}$ **C)** $\frac{3}{5}$ **D)** $\frac{4}{5}$ **E)** 1

**Q15**. Rectangle $ABCD$ has $AB = 3$ and $BC = 4$. Point $E$ is the foot of the perpendicular from $B$ to diagonal $\overline{AC}$. What is the area of $\triangle AED$?

**A)** 1 **B)** $\frac{42}{25}$ **C)** $\frac{28}{15}$ **D)** 2 **E)** $\frac{54}{25}$

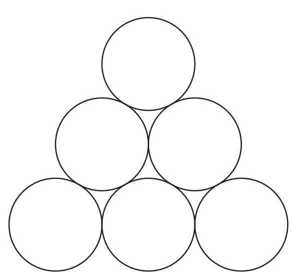
**Q16**. How many of the base-ten numerals for the positive integers less than or equal to 2017 contain the digit 0?

**A)** 469 **B)** 471 **C)** 475 **D)** 478 **E)** 481

**Q17**. Call a positive integer *monotonous* if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

**A)** 1024 **B)** 1524 **C)** 1533 **D)** 1536 **E)** 2048

**Q18**. In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?



**A)** 6 **B)** 8 **C)** 9 **D)** 12 **E)** 15

**Q19**. Let $ABC$ be an equilateral triangle. Extend side $\overline{AB}$ beyond $B$ to a point $B'$ so that $BB' = 3 \cdot AB$. Similarly, extend side $\overline{BC}$ beyond $C$ to a point $C'$ so that $CC' = 3 \cdot BC$, and extend side $\overline{CA}$ beyond $A$ to a point $A'$ so that $AA' = 3 \cdot CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

**A)** 9 **B)** 16 **C)** 25 **D)** 36 **E)** 37

**Q20**. The number $21! = 51,090,942,171,709,440,000$ has over $60,000$ positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

**A)** $\frac{1}{21}$ **B)** $\frac{1}{19}$ **C)** $\frac{1}{18}$ **D)** $\frac{1}{2}$ **E)** $\frac{11}{21}$

**Q21**. In $\triangle ABC$, $AB = 6$, $AC = 8$, $BC = 10$, and $D$ is the midpoint of $\overline{BC}$. What is the sum of the radii of the circles inscribed in $\triangle ADB$ and $\triangle ADC$?

**A)** $\sqrt{5}$ **B)** $\frac{11}{4}$ **C)** $2\sqrt{2}$ **D)** $\frac{17}{6}$ **E)** 3

**Q22**. The diameter $\overline{AB}$ of a circle of radius 2 is extended to a point $D$ outside the circle so that $BD = 3$. Point $E$ is chosen so that $ED = 5$ and line $ED$ is perpendicular to line $AD$. Segment $\overline{AE}$ intersects the circle at a point $C$ between $A$ and $E$. What is the area of $\triangle ABC$?

**A)** $\frac{120}{37}$ **B)** $\frac{140}{39}$ **C)** $\frac{145}{39}$ **D)** $\frac{140}{37}$ **E)** $\frac{120}{31}$

**Q23**. Let $N = 123456789101112\dots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when $N$ is divided by 45?

**A)** 1 **B)** 4 **C)** 9 **D)** 18 **E)** 44

**Q24**. The vertices of an equilateral triangle lie on the hyperbola $xy = 1$, and a vertex of this hyperbola is the centroid of the triangle. What is the square of the area of the triangle?

**A)** 48 **B)** 60 **C)** 108 **D)** 120 **E)** 169

**Q25**. Last year Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. What was her score on the sixth test?

**A)** 92 **B)** 94 **C)** 96 **D)** 98 **E)** 100

**Q1**. What is the value of $\dfrac{11! - 10!}{9!}$?

**A)** 99 **B)** 100 **C)** 110 **D)** 121 **E)** 132

**Q2**. For what value of $x$ does $10^x \cdot 100^{2x} = 1000^5$?

**A)** 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

**Q3**. For every dollar Ben spent on bagels, David spent 25 cents less. Ben paid \$12.50 more than David. How much did they spend in the bagel store together?

**A)** \$37.50 **B)** \$50.00 **C)** \$87.50 **D)** \$90.00 **E)** \$92.50

**Q4**. The remainder can be defined for all real numbers $x$ and $y$ with $y \neq 0$ by

$$\text{rem}(x, y) = x - y \left\lfloor \frac{x}{y} \right\rfloor$$

where $\left\lfloor \frac{x}{y} \right\rfloor$ denotes the greatest integer less than or equal to $\frac{x}{y}$. What is the value of $\text{rem}(\frac{3}{8}, -\frac{2}{5})$?

**A)** $-\dfrac{3}{8}$ **B)** $-\dfrac{1}{40}$ **C)** 0 **D)** $\dfrac{3}{8}$ **E)** $\dfrac{31}{40}$

**Q5**. A rectangular box has integer side lengths in the ratio $1 : 3 : 4$. Which of the following could be the volume of the box?

**A)** 48 **B)** 56 **C)** 64 **D)** 96 **E)** 144

**Q6**. Ximena lists the whole numbers 1 through 30 once. Emilio copies Ximena's numbers, replacing each occurrence of the digit 2 by the digit 1. Ximena adds her numbers and Emilio adds his numbers. How much larger is Ximena's sum than Emilio's?

**A)** 13 **B)** 26 **C)** 102 **D)** 103 **E)** 110

**Q7**. The mean, median, and mode of the 7 data values $60, 100, x, 40, 50, 200, 90$ are all equal to $x$. What is the value of $x$?

**A)** 50 **B)** 60 **C)** 75 **D)** 90 **E)** 100

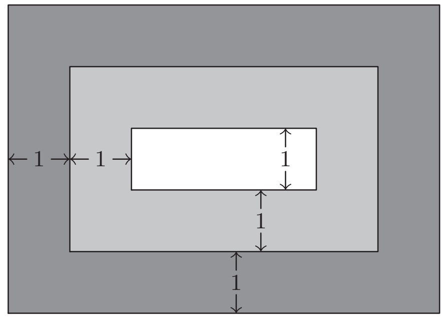
**Q8**. Trickster Rabbit agrees with Foolish Fox to double Fox's money every time Fox crosses the bridge by Rabbit's house, as long as Fox pays 40 coins in toll to Rabbit after each crossing. The payment is made after the doubling, Fox is excited about his good fortune until he discovers that all his money is gone after crossing the bridge three times. How many coins did Fox have at the beginning?

**A)** 20 **B)** 30 **C)** 35 **D)** 40 **E)** 45

**Q9**. A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to $N$ coins in the $N$th row. What is the sum of the digits of $N$?

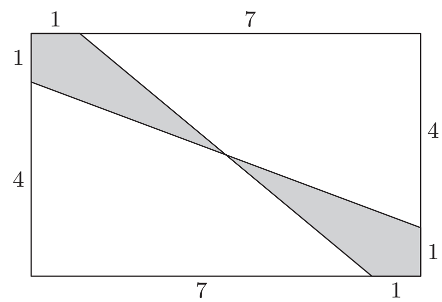
**A)** 6 **B)** 7 **C)** 8 **D)** 9 **E)** 10

**Q10**. A rug is made with three different colors as shown. The areas of the three differently colored regions form an arithmetic progression. The inner rectangle is one foot wide, and each of the two shaded regions is 1 foot wide on all four sides. What is the length in feet of the inner rectangle?




**A)** 1 **B)** 2 **C)** 4 **D)** 6 **E)** 8

**Q11**. Find the area of the shaded region.




**A)** $4\frac{3}{5}$ **B)** 5 **C)** $5\frac{1}{4}$ **D)** $6\frac{1}{2}$ **E)** 8

**Q12**. Three distinct integers are selected at random between 1 and 2016, inclusive. Which of the following is a correct statement about the probability $p$ that the product of the three integers is odd?

**A)** $p < \frac{1}{8}$ **B)** $p = \frac{1}{8}$ **C)** $\frac{1}{8} < p < \frac{1}{3}$ **D)** $p = \frac{1}{3}$ **E)** $p > \frac{1}{3}$

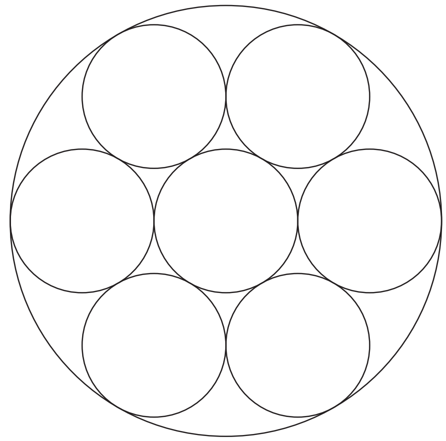
**Q13**. Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions "left" and "right" are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

**A)** 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

**Q14**. How many ways are there to write 2016 as the sum of twos and threes, ignoring order? (For example, $1008 \cdot 2 + 0 \cdot 3$ and $402 \cdot 2 + 404 \cdot 3$ are two such ways.)

**A)** 236 **B)** 336 **C)** 337 **D)** 403 **E)** 672

**Q15**. Seven cookies of radius 1 inch are cut from a circle of cookie dough, as shown. Neighboring cookies are tangent, and all except the center cookie are tangent to the edge of the dough. The leftover scrap is reshaped to form another cookie of the same thickness. What is the radius in inches of the scrap cookie?



**A)** $\sqrt{2}$ **B)** 1.5 **C)** $\sqrt{\pi}$ **D)** $\sqrt{2\pi}$ **E)** $\pi$

**Q16**. A triangle with vertices $A(0,2)$, $B(-3,2)$, and $C(-3,0)$ is reflected about the $x$-axis, then the image $\triangle A'B'C'$ is rotated counterclockwise about the origin by $90^\circ$ to produce $\triangle A''B''C''$. Which of the following transformations will return $\triangle A''B''C''$ to $\triangle ABC$?

**A)** counterclockwise rotation about the origin by $90^\circ$.

**B)** clockwise rotation about the origin by $90^\circ$.

**C)** reflection about the $x$-axis

**D)** reflection about the line $y = x$

**E)** reflection about the $y$-axis.

**Q17**. Let $N$ be a positive multiple of 5. One red ball and $N$ green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as $N$ grows large. What is the sum of the digits of the least value of $N$ such that $P(N) < \frac{321}{400}$?

**A)** 12 **B)** 14 **C)** 16 **D)** 18 **E)** 20

**Q18**. Each vertex of a cube is to be labeled with an integer 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

**A)** 1 **B)** 3 **C)** 6 **D)** 12 **E)** 24

**Q19**. In rectangle $ABCD$, $AB = 6$ and $BC = 3$. Point $E$ between $B$ and $C$, and point $F$ between $E$ and $C$ are such that $BE = EF = FC$. Segments $\overline{AE}$ and $\overline{AF}$ intersect $\overline{BD}$ at $P$ and $Q$, respectively. The ratio $BP : PQ : QD$ can be written as $r : s : t$ where the greatest common factor of $r, s,$ and $t$ is 1. What is $r + s + t$?

**A)** 7 **B)** 9 **C)** 12 **D)** 15 **E)** 20

**Q20**. For some particular value of $N$, when $(a + b + c + d + 1)^N$ is expanded and like terms are combined, the resulting expression contains exactly 1001 terms that include all four variables $a, b, c,$ and $d$, each to some positive power. What is $N$?

**A)** 9 **B)** 14 **C)** 16 **D)** 17 **E)** 19

**Q21**. Circles with centers $P, Q$ and $R$, having radii $1, 2$ and $3$, respectively, lie on the same side of line $l$ and are tangent to $l$ at $P', Q'$ and $R'$, respectively, with $Q'$ between $P'$ and $R'$. The circle with center $Q$ is externally tangent to each of the other two circles. What is the area of triangle $PQR$?

**A)** 0 **B)** $\sqrt{6}/3$ **C)** 1 **D)** $\sqrt{6}-\sqrt{2}$ **E)** $\sqrt{6}/2$

**Q22**. For some positive integer $n$, the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

**A)** 110 **B)** 191 **C)** 261 **D)** 325 **E)** 425

**Q23**. A binary operation $\diamondsuit$ has the properties that $a \diamondsuit (b \diamondsuit c) = (a \diamondsuit b) \cdot c$ and that $a \diamondsuit a = 1$ for all nonzero real numbers $a, b$, and $c$. (Here $\cdot$ represents multiplication). The solution to the equation $2016 \diamondsuit (6 \diamondsuit x) = 100$ can be written as $\frac{p}{q}$, where $p$ and $q$ are relatively prime positive integers. What is $p+q$?

**A)** 109 **B)** 201 **C)** 301 **D)** 3049 **E)** $33,601$

**Q24**. A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of the fourth side?

**A)** 200 **B)** $200\sqrt{2}$ **C)** $200\sqrt{3}$ **D)** $300\sqrt{2}$ **E)** 500

**Q25**. How many ordered triples $(x, y, z)$ of positive integers satisfy $\text{lcm}(x, y) = 72, \text{lcm}(x, z) = 600$ and $\text{lcm}(y, z) = 900$?

**A)** 15 **B)** 16 **C)** 24 **D)** 27 **E)** 64

**Q1**. What is the value of $\dfrac{2a^{-1}+\dfrac{a^{-1}}{2}}{a}$ when $a=\frac{1}{2}$?

**A)** 1 **B)** 2 **C)** $\dfrac{5}{2}$ **D)** 10 **E)** 20

**Q2**. If $n\heartsuit m=n^3m^2$, what is $\dfrac{2\heartsuit 4}{4\heartsuit 2}$?

**A)** $\dfrac{1}{4}$ **B)** $\dfrac{1}{2}$ **C)** 1 **D)** 2 **E)** 4

**Q3**. Let $x=-2016$. What is the value of

$$\Bigg|\big||x|-x\big|-|x|\Bigg|-x?$$

**A)** $-2016$ **B)** 0 **C)** 2016 **D)** 4032 **E)** 6048

**Q4**. Zoey read 15 books, one at a time. The first book took her 1 day to read, the second book took her 2 days to read, the third book took her 3 days to read, and so on, with each book taking her 1 more day to read than the previous book. Zoey finished the first book on a Monday, and the second on a Wednesday. On what day the week did she finish her 15th book?

**A)** Sunday **B)** Monday **C)** Wednesday **D)** Friday **E)** Saturday

**Q5**. The mean age of Amanda's 4 cousins is 8, and their median age is 5. What is the sum of the ages of Amanda's youngest and oldest cousins?

**A)** 13 **B)** 16 **C)** 19 **D)** 22 **E)** 25

**Q6**. Laura added two three-digit positive integers. All six digits in these numbers are different. Laura's sum is a three-digit number $S$. What is the smallest possible value for the sum of the digits of $S$?

**A)** 1 **B)** 4 **C)** 5 **D)** 15 **E)** 20

**Q7**. The ratio of the measures of two acute angles is $5:4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

**A)** 75 **B)** 90 **C)** 135 **D)** 150 **E)** 270

**Q8**. What is the tens digit of $2015^{2016}-2017$?

**A)** 0 **B)** 1 **C)** 3 **D)** 5 **E)** 8

**Q9**. All three vertices of $\triangle ABC$ lie on the parabola defined by $y=x^2$, with $A$ at the origin and $\overline{BC}$ parallel to the $x$-axis. The area of the triangle is 64. What is the length of $BC$?

**A)** 4 **B)** 6 **C)** 8 **D)** 10 **E)** 16

**Q10**. A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

**A)** 14.0 **B)** 16.0 **C)** 20.0 **D)** 33.3 **E)** 55.6

**Q11**. Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden?

**A)** 256 **B)** 336 **C)** 384 **D)** 448 **E)** 512

**Q12**. Two different numbers are selected at random from $\{1, 2, 3, 4, 5\}$ and multiplied together. What is the probability that the product is even?

**A)** 0.2 **B)** 0.4 **C)** 0.5 **D)** 0.7 **E)** 0.8

**Q13**. At Megapolis Hospital one year, multiple-birth statistics were as follows: Sets of twins, triplets, and quadruplets accounted for 1000 of the babies born. There were four times as many sets of triplets as sets of quadruplets, and there was three times as many sets of twins as sets of triplets. How many of these 1000 babies were in sets of quadruplets?

**A)** 25 **B)** 40 **C)** 64 **D)** 100 **E)** 160

**Q14**. How many squares whose sides are parallel to the axes and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y = \pi x$, the line $y = -0.1$ and the line $x = 5.1$?

**A)** 30 **B)** 41 **C)** 45 **D)** 50 **E)** 57

**Q15**. All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a $3 \times 3$ array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What is the number in the center?

**A)** 5 **B)** 6 **C)** 7 **D)** 8 **E)** 9

**Q16**. The sum of an infinite geometric series is a positive number $S$, and the second term in the series is 1. What is the smallest possible value of $S$?

**A)** $\frac{1+\sqrt{5}}{2}$ **B)** 2 **C)** $\sqrt{5}$ **D)** 3 **E)** 4

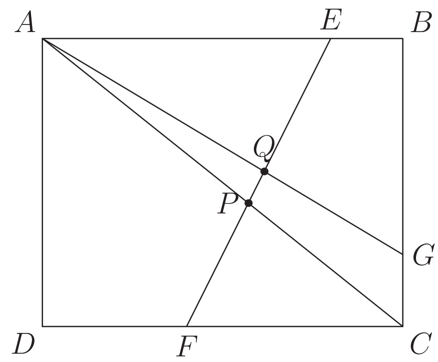
**Q17**. All the numbers $2, 3, 4, 5, 6, 7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

**A)** 312 **B)** 343 **C)** 625 **D)** 729 **E)** 1680

**Q18**. In how many ways can 345 be written as the sum of an increasing sequence of two or more consecutive positive integers?

**A)** 1 **B)** 3 **C)** 5 **D)** 6 **E)** 7

**Q19**. Rectangle $ABCD$ has $AB = 5$ and $BC = 4$. Point $E$ lies on $\overline{AB}$ so that $EB = 1$, point $G$ lies on $\overline{BC}$ so that $CG = 1$, and point $F$ lies on $\overline{CD}$ so that $DF = 2$. Segments $\overline{AG}$ and $\overline{AC}$ intersect $\overline{EF}$ at $Q$ and $P$, respectively. What is the value of $\frac{PQ}{EF}$?




**A)** $\frac{\sqrt{13}}{16}$ **B)** $\frac{\sqrt{2}}{13}$ **C)** $\frac{9}{82}$ **D)** $\frac{10}{91}$ **E)** $\frac{1}{9}$

**Q20**. A dilation of the plane—that is, a size transformation with a positive scale factor—sends the circle of radius 2 centered at $A(2,2)$ to the circle of radius 3 centered at $A$ß$(5,6)$. What distance does the origin $O(0,0)$, move under this transformation?

**A)** 0 **B)** 3 **C)** $\sqrt{13}$ **D)** 4 **E)** 5

**Q21**. What is the area of the region enclosed by the graph of the equation $x^2 + y^2 = |x| + |y|$?

**A)** $\pi + \sqrt{2}$ **B)** $\pi + 2$ **C)** $\pi + 2\sqrt{2}$ **D)** $2\pi + \sqrt{2}$ **E)** $2\pi + 2\sqrt{2}$

**Q22**. A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A, B, C\}$ were there in which $A$ beat $B$, $B$ beat $C$, and $C$ beat $A$?

**A)** 385 **B)** 665 **C)** 945 **D)** 1140 **E)** 1330

**Q23**. In regular hexagon $ABCDEF$, points $W$, $X$, $Y$, and $Z$ are chosen on sides $\overline{BC}$, $\overline{CD}$, $\overline{EF}$, and $\overline{FA}$ respectively, so lines $AB$, $ZW$, $YX$, and $ED$ are parallel and equally spaced. What is the ratio of the area of hexagon $WCXYFZ$ to the area of hexagon $ABCDEF$?

**A)** $\frac{1}{3}$ **B)** $\frac{10}{27}$ **C)** $\frac{11}{27}$ **D)** $\frac{4}{9}$ **E)** $\frac{13}{27}$

**Q24**. How many four-digit integers $abcd$, with $a \neq 0$, have the property that the three two-digit integers $ab < bc < cd$ form an increasing arithmetic sequence? One such number is 4692, where $a = 4$, $b = 6$, $c = 9$, and $d = 2$.

**A)** 9 **B)** 15 **C)** 16 **D)** 17 **E)** 20

**Q25**. Let $f(x) = \sum_{k=2}^{10}(\lfloor kx \rfloor - k\lfloor x \rfloor)$, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to $r$. How many distinct values does $f(x)$ assume for $x \geq 0$?

**A)** 32 **B)** 36 **C)** 45 **D)** 46 **E)** infinitely many

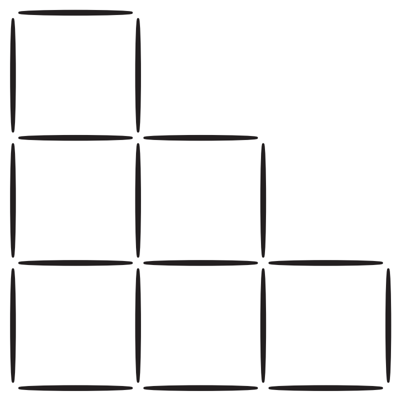
**Q1**. What is the value of $(2^0 - 1 + 5^2 - 0)^{-1} \times 5$?

**A)** $-125$ **B)** $-120$ **C)** $\frac{1}{5}$ **D)** $\frac{5}{24}$ **E)** $25$

**Q2**. A box contains a collection of triangular and square tiles. There are 25 tiles in the box, containing 84 edges total. How many square tiles are there in the box?

**A)** 3 **B)** 5 **C)** 7 **D)** 9 **E)** 11

**Q3**. Ann made a 3-step staircase using 18 toothpicks as shown in the figure. How many toothpicks does she need to add to complete a 5-step staircase?



**A)** 9 **B)** 18 **C)** 20 **D)** 22 **E)** 24

**Q4**. Pablo, Sofia, and Mia got some candy eggs at a party. Pablo had three times as many eggs as Sofia, and Sofia had twice as many eggs as Mia. Pablo decides to give some of his eggs to Sofia and Mia so that all three will have the same number of eggs. What fraction of his eggs should Pablo give to Sofia?

**A)** $\frac{1}{12}$ **B)** $\frac{1}{6}$ **C)** $\frac{1}{4}$ **D)** $\frac{1}{3}$ **E)** $\frac{1}{2}$

**Q5**. Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the test average became 81. What was Payton's score on the test?

**A)** 81 **B)** 85 **C)** 91 **D)** 94 **E)** 95

**Q6**. The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller number?

**A)** $\frac{5}{4}$ **B)** $\frac{3}{2}$ **C)** $\frac{9}{5}$ **D)** 2 **E)** $\frac{5}{2}$

**Q7**. How many terms are in the arithmetic sequence 13, 16, 19, . . . , 70, 73?

**A)** 20 **B)** 21 **C)** 24 **D)** 60 **E)** 61

**Q8**. Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2 : 1$ ?

**A)** 2 **B)** 4 **C)** 5 **D)** 6 **E)** 8

**Q9**. Two right circular cylinders have the same volume. The radius of the second cylinder is 10% more than the radius of the first. What is the relationship between the heights of the two cylinders?

**A)** The second height is 10% less than the first.

**B)** The first height is 10% more than the second.

**C)** The second height is 21% less than the first.

**D)** The first height is 21% more than the second.

**E)** The second height is 80% of the first.

**Q10**. How many rearrangements of *abcd* are there in which no two adjacent letters are also adjacent letters in the alphabet? For example, no such rearrangements could include either *ab* or *ba*.

**A)** 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

**Q11**. The ratio of the length to the width of a rectangle is $4:3$. If the rectangle has diagonal of length $d$, then the area may be expressed as $kd^2$ for some constant $k$. What is $k$?

**A)** $\frac{2}{7}$ **B)** $\frac{3}{7}$ **C)** $\frac{12}{25}$ **D)** $\frac{16}{25}$ **E)** $\frac{3}{4}$

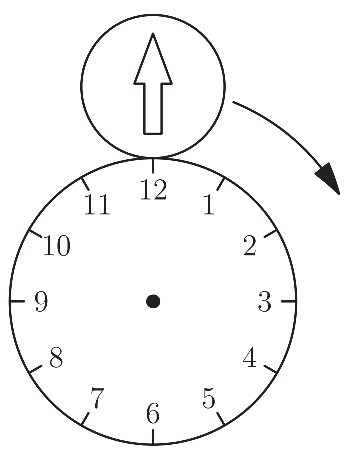
**Q12**. Points $(\sqrt{\pi}, a)$ and $(\sqrt{\pi}, b)$ are distinct points on the graph of $y^2 + x^4 = 2x^2y + 1$. What is $|a-b|$?

**A)** 1 **B)** $\frac{\pi}{2}$ **C)** 2 **D)** $\sqrt{1+\pi}$ **E)** $1+\sqrt{\pi}$

**Q13**. Claudia has 12 coins, each of which is a 5-cent coin or a 10-cent coin. There are exactly 17 different values that can be obtained as combinations of one or more of her coins. How many 10-cent coins does Claudia have?

**A)** 3 **B)** 4 **C)** 5 **D)** 6 **E)** 7

**Q14**. The diagram below shows the circular face of a clock with radius 20 cm and a circular disk with radius 10 cm externally tangent to the clock face at 12 o' clock. The disk has an arrow painted on it, initially pointing in the upward vertical direction. Let the disk roll clockwise around the clock face. At what point on the clock face will the disk be tangent when the arrow is next pointing in the upward vertical direction?




**A)** 2 o'clock **B)** 3 o'clock **C)** 4 o'clock **D)** 6 o'clock **E)** 8 o'clock

**Q15**. Consider the set of all fractions $\frac{x}{y}$, where $x$ and $y$ are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

**A)** 0 **B)** 1 **C)** 2 **D)** 3 **E)** infinitely many

**Q16**. If $y+4=(x-2)^2, x+4=(y-2)^2$, and $x \neq y$, what is the value of $x^2+y^2$?

**A)** 10 **B)** 15 **C)** 20 **D)** 25 **E)** 30

**Q17**. A line that passes through the origin intersects both the line $x = 1$ and the line $y = 1 + \frac{\sqrt{3}}{3}x$. The three lines create an equilateral triangle. What is the perimeter of the triangle?

**A)** $2\sqrt{6}$ **B)** $2 + 2\sqrt{3}$ **C)** 6 **D)** $3 + 2\sqrt{3}$ **E)** $6 + \frac{\sqrt{3}}{3}$

**Q18**. Hexadecimal (base-16) numbers are written using numeric digits 0 through 9 as well as the letters $A$ through $F$ to represent 10 through 15. Among the first 1000 positive integers, there are $n$ whose hexadecimal representation contains only numeric digits. What is the sum of the digits of $n$?

**A)** 17 **B)** 18 **C)** 19 **D)** 20 **E)** 21

**Q19**. The isosceles right triangle $ABC$ has right angle at $C$ and area 12.5. The rays trisecting $\angle ACB$ intersect $AB$ at $D$ and $E$. What is the area of $\triangle CDE$?

**A)** $\frac{5\sqrt{2}}{3}$ **B)** $\frac{50\sqrt{3} - 75}{4}$ **C)** $\frac{15\sqrt{3}}{8}$ **D)** $\frac{50 - 25\sqrt{3}}{2}$ **E)** $\frac{25}{6}$

**Q20**. A rectangle with positive integer side lengths in cm has area $A$ cm$^2$ and perimeter $P$ cm. Which of the following numbers cannot equal $A + P$?

**A)** 100 **B)** 102 **C)** 104 **D)** 106 **E)** 108

**Q21**. Tetrahedron $ABCD$ has $AB = 5$, $AC = 3$, $BC = 4$, $BD = 4$, $AD = 3$, and $CD = \frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

**A)** $3\sqrt{2}$ **B)** $2\sqrt{5}$ **C)** $\frac{24}{5}$ **D)** $3\sqrt{3}$ **E)** $\frac{24}{5}\sqrt{2}$

**Q22**. Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

**A)** $\frac{47}{256}$ **B)** $\frac{3}{16}$ **C)** $\frac{49}{256}$ **D)** $\frac{25}{128}$ **E)** $\frac{51}{256}$

**Q23**. The zeroes of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of $a$?

**A)** 7 **B)** 8 **C)** 16 **D)** 17 **E)** 18

**Q24**. For some positive integers $p$, there is a quadrilateral $ABCD$ with positive integer side lengths, perimeter $p$, right angles at $B$ and $C$, $AB = 2$, and $CD = AD$. How many different values of $p < 2015$ are possible?

**A)** 30 **B)** 31 **C)** 61 **D)** 62 **E)** 63

**Q25**. Let $S$ be a square of side length 1. Two points are chosen independently at random on the sides of $S$. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a - b\pi}{c}$, where $a$, $b$, and $c$ are positive integers with $\gcd(a, b, c) = 1$. What is $a + b + c$?

**A)** 59 **B)** 60 **C)** 61 **D)** 62 **E)** 63

**Q1**. What is the value of $2-(-2)^{-2}$?

**A)** $-2$ **B)** $\frac{1}{16}$ **C)** $\frac{7}{4}$ **D)** $\frac{9}{4}$ **E)** $6$

**Q2**. Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

**A)** 3:10 PM **B)** 3:30 PM **C)** 4:00 PM **D)** 4:10 PM **E)** 4:30 PM

**Q3**. Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

**A)** 8 **B)** 11 **C)** 14 **D)** 15 **E)** 18

**Q4**. Four siblings ordered an extra large pizza. Alex ate $\frac{1}{5}$, Beth $\frac{1}{3}$, and Cyril $\frac{1}{4}$ of the pizza. Dan got the leftovers. What is the sequence of the siblings in decreasing order of the part of pizza they consumed?

**A)** Alex, Beth, Cyril, Dan

**B)** Beth, Cyril, Alex, Dan

**C)** Beth, Cyril, Dan, Alex

**D)** Beth, Dan, Cyril, Alex

**E)** Dan, Beth, Cyril, Alex

**Q5**. David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

**A)** David **B)** Hikmet **C)** Jack **D)** Rand **E)** Todd

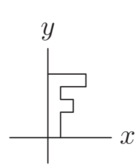
**Q6**. Marley practices exactly one sport each day of the week. She runs three days a week but never on two consecutive days. On Monday she plays basketball and two days later golf. She swims and plays tennis, but she never plays tennis the day after running or swimming. Which day of the week does Marley swim?

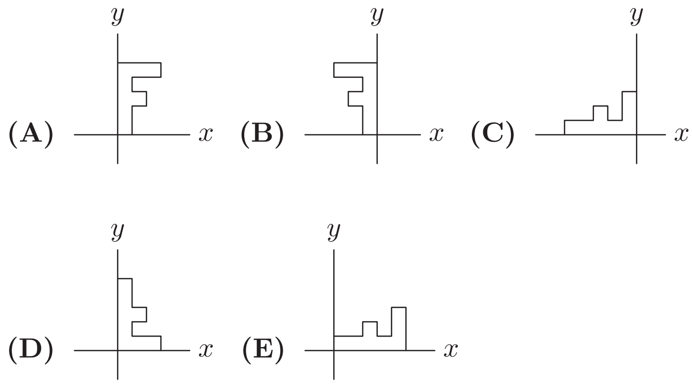
**A)** Sunday **B)** Tuesday **C)** Thursday **D)** Friday **E)** Saturday

**Q7**. Consider the operation "minus the reciprocal of," defined by $a\diamond b=a-\frac{1}{b}$. What is $((1\diamond 2)\diamond 3)-(1\diamond(2\diamond 3))$?

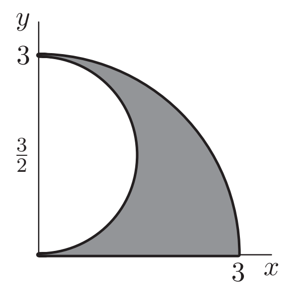
**A)** $-\frac{7}{30}$ **B)** $-\frac{1}{6}$ **C)** $0$ **D)** $\frac{1}{6}$ **E)** $\frac{7}{30}$

**Q8**. The letter $F$ shown below is rotated $90^\circ$ clockwise around the origin, then reflected in the $y$-axis, and then rotated a half turn around the origin. What is the final image?






**Q9**. The shaded region below is called a shark's fin falcata, a figure studied by Leonardo da Vinci. It is bounded by the portion of the circle of radius 3 and center $(0, 0)$ that lies in the first quadrant, the portion of the circle with radius $\frac{3}{2}$ and center $(0, \frac{3}{2})$ that lies in the first quadrant, and the line segment from $(0, 0)$ to $(3, 0)$. What is the area of the shark's fin falcata?




**A)** $\frac{4\pi}{5}$ **B)** $\frac{9\pi}{8}$ **C)** $\frac{4\pi}{3}$ **D)** $\frac{7\pi}{5}$ **E)** $\frac{3\pi}{2}$

**Q10**. What are the sign and units digit of the product of all the odd negative integers strictly greater than $-2015$?

**A)** It is a negative number ending with a 1.

**B)** It is a positive number ending with a 1.

**C)** It is a negative number ending with a 5.

**D)** It is a positive number ending with a 5.

**E)** It is a negative number ending with a 0.

**Q11**. Among the positive integers less than 100, each of whose digits is a prime number, one is selected at random. What is the probability that the selected number is prime?

**A)** $\frac{8}{99}$ **B)** $\frac{2}{5}$ **C)** $\frac{9}{20}$ **D)** $\frac{1}{2}$ **E)** $\frac{9}{16}$

**Q12**. For how many integers $x$ is the point $(x, -x)$ inside or on the circle of radius 10 centered at $(5, 5)$?

**A)** 11 **B)** 12 **C)** 13 **D)** 14 **E)** 15

**Q13**. The line $12x + 5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

**A)** 20 **B)** $\frac{360}{17}$ **C)** $\frac{107}{5}$ **D)** $\frac{43}{2}$ **E)** $\frac{281}{13}$

**Q14**. Let $a$, $b$, and $c$ be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x - a)(x - b) + (x - b)(x - c) = 0$?

**A)** 15 **B)** 15.5 **C)** 16 **D)** 16.5 **E)** 17

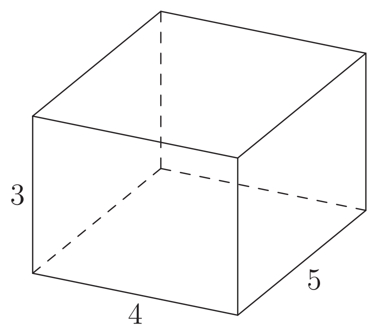
**Q15**. The town of Hamlet has 3 people for each horse, 4 sheep for each cow, and 3 ducks for each person. Which of the following could not possibly be the total number of people, horses, sheep, cows, and ducks in Hamlet?

**A)** 41 **B)** 47 **C)** 59 **D)** 61 **E)** 66

**Q16**. Al, Bill, and Cal will each randomly be assigned a whole number from 1 to 10, inclusive, with no two of them getting the same number. What is the probability that Al's number will be a whole number multiple of Bill's and Bill's number will be a whole number multiple of Cal's?

**A)** $\frac{9}{1000}$ **B)** $\frac{1}{90}$ **C)** $\frac{1}{80}$ **D)** $\frac{1}{72}$ **E)** $\frac{2}{121}$

**Q17**. The centers of the faces of the right rectangular prism shown below are joined to create an octahedron. What is the volume of this octahedron?




**A)** $\frac{75}{12}$ **B)** 10 **C)** 12 **D)** $10\sqrt{2}$ **E)** 15

**Q18**. Johann has 64 fair coins. He flips all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. What is the expected number of coins that are now heads?

**A)** 32 **B)** 40 **C)** 48 **D)** 56 **E)** 64

**Q19**. In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points $X, Y, Z$, and $W$ lie on a circle. What is the perimeter of the triangle?

**A)** $12 + 9\sqrt{3}$ **B)** $18 + 6\sqrt{3}$ **C)** $12 + 12\sqrt{2}$ **D)** 30 **E)** 32

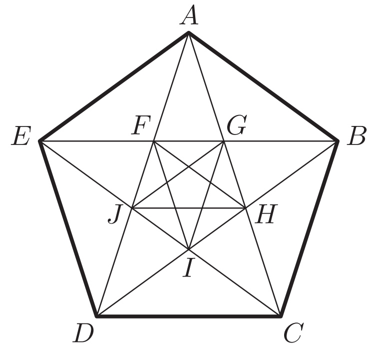
**Q20**. Erin the ant starts at a given corner of a cube and crawls along exactly 7 edges in such a way that she visits every corner exactly once and then finds that she is unable to return along an edge to her starting point. How many paths are there meeting these conditions?

**A)** 6 **B)** 9 **C)** 12 **D)** 18 **E)** 24

**Q21**. Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose the Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let $s$ denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of $s$?

**A)** 9 **B)** 11 **C)** 12 **D)** 13 **E)** 15

**Q22**. In the figure shown below, $ABCDE$ is a regular pentagon and $AG = 1$. What is $FG + JH + CD$?




**A)** 3 **B)** $12-4\sqrt{5}$ **C)** $\frac{5+2\sqrt{5}}{3}$ **D)** $1+\sqrt{5}$ **E)** $\frac{11+11\sqrt{5}}{10}$

**Q23**. Let $n$ be a positive integer greater than 4 such that the decimal representation of $n!$ ends in $k$ zeros and the decimal representation of $(2n)!$ ends in $3k$ zeros. Let $s$ denote the sum of the four least possible values of $n$. What is the sum of the digits of $s$?

**A)** 7 **B)** 8 **C)** 9 **D)** 10 **E)** 11

**Q24**. Aaron the ant walks on the coordinate plane according to the following rules. He starts at the origin $p_0 = (0, 0)$ facing to the east and walks one unit, arriving at $p_1 = (1, 0)$. For $n = 1, 2, 3, \ldots$, right after arriving at the point $p_n$, if Aaron can turn 90° left and walk one unit to an unvisited point $p_{n+1}$, he does that. Otherwise, he walks one unit straight ahead to reach $p_{n+1}$. Thus the sequence of points continues $p_2 = (1, 1), p_3 = (0, 1), p_4 = (-1, 1), p_5 = (-1, 0)$, and so on in a counterclockwise spiral pattern. What is $p_{2015}$?

**A)** $(-22, -13)$ **B)** $(-13, -22)$ **C)** $(-13, 22)$ **D)** $(13, -22)$ **E)** $(22, -13)$

**Q25**. A rectangular box measures $a \times b \times c$, where $a$, $b$, and $c$ are integers and $1 \le a \le b \le c$. The volume and the surface area of the box are numerically equal. How many ordered triples $(a, b, c)$ are possible?

**A)** 4 **B)** 10 **C)** 12 **D)** 21 **E)** 26

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7. Nonzero real numbers $x$, $y$, $a$, and $b$ satisfy $x < a$ and $y < b$. How many of the following inequalities must be true?

(I) $x + y < a + b$
(II) $x - y < a - b$
(III) $xy < ab$
(IV) $\frac{x}{y} < \frac{a}{b}$

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

8. Which of the following numbers is a perfect square?

**(A)** $\dfrac{14!15!}{2}$ **(B)** $\dfrac{15!16!}{2}$ **(C)** $\dfrac{16!17!}{2}$ **(D)** $\dfrac{17!18!}{2}$ **(E)** $\dfrac{18!19!}{2}$

9. The two legs of a right triangle, which are altitudes, have lengths $2\sqrt{3}$ and 6. How long is the third altitude of the triangle?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

10. Five positive consecutive integers starting with $a$ have average $b$. What is the average of 5 consecutive integers that start with $b$?

**(A)** $a + 3$ **(B)** $a + 4$ **(C)** $a + 5$ **(D)** $a + 6$ **(E)** $a + 7$

11. A customer who intends to purchase an appliance has three coupons, only one of which may be used:

Coupon 1: 10% off the listed price if the listed price is at least \$50

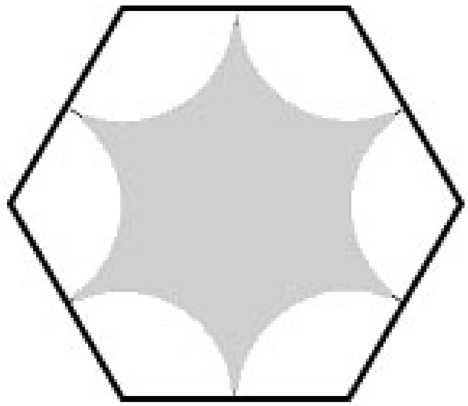
Coupon 2: \$20 off the listed price if the listed price is at least \$100

Coupon 3: 18% off the amount by which the listed price exceeds \$100

For which of the following listed prices will coupon 1 offer a greater price reduction than either coupon 2 or coupon 3?

**(A)** \$179.95 **(B)** \$199.95 **(C)** \$219.95 **(D)** \$239.95 **(E)** \$259.95

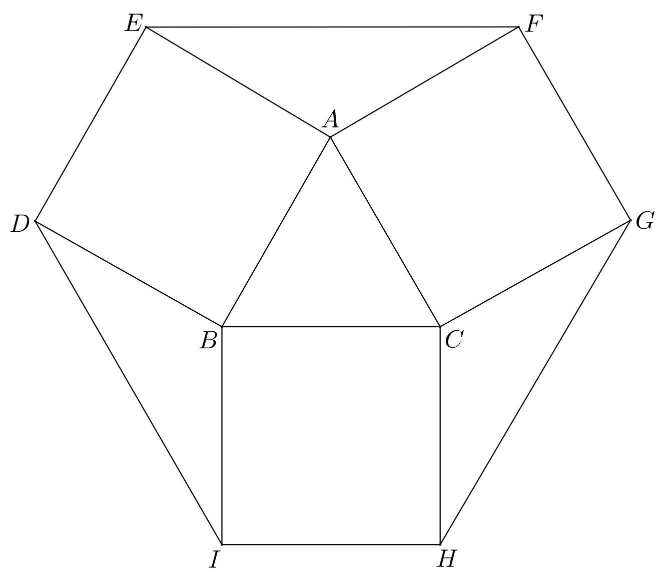
12. A regular hexagon has side length 6. Congruent arcs with radius 3 are drawn with the center at each of the vertices, creating circular sectors as shown. The region inside the hexagon but outside the sectors is shaded as shown. What is the area of the shaded region?



**(A)** $27\sqrt{3} - 9\pi$ **(B)** $27\sqrt{3} - 6\pi$ **(C)** $54\sqrt{3} - 18\pi$

**(D)** $54\sqrt{3} - 12\pi$ **(E)** $108\sqrt{3} - 9\pi$

13. Equilateral $\triangle ABC$ has side length 1, and squares $ABDE$, $BCHI$, and $CAFG$ lie outside the triangle. What is the area of hexagon $DEFGHI$?




**(A)** $\frac{12+3\sqrt{3}}{4}$ **(B)** $\frac{9}{2}$ **(C)** $3+\sqrt{3}$ **(D)** $\frac{6+3\sqrt{3}}{2}$ **(E)** 6

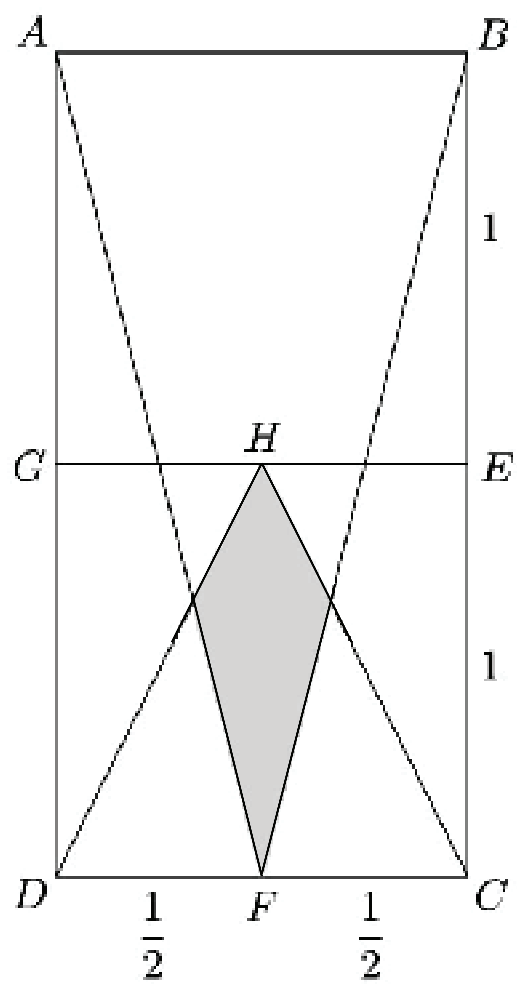
14. The $y$-intercepts, $P$ and $Q$, of two perpendicular lines intersecting at the point $A(6, 8)$ have a sum of zero. What is the area of $\triangle APQ$?

**(A)** 45 **(B)** 48 **(C)** 54 **(D)** 60 **(E)** 72

15. David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

**(A)** 140 **(B)** 175 **(C)** 210 **(D)** 245 **(E)** 280

16. In rectangle $ABCD$, $AB = 1$, $BC = 2$, and points $E$, $F$, and $G$ are midpoints of $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$, respectively. Point $H$ is the midpoint of $\overline{GE}$. What is the area of the shaded region?




**(A)** $\frac{1}{12}$ **(B)** $\frac{\sqrt{3}}{18}$ **(C)** $\frac{\sqrt{2}}{12}$ **(D)** $\frac{\sqrt{3}}{12}$ **(E)** $\frac{1}{6}$

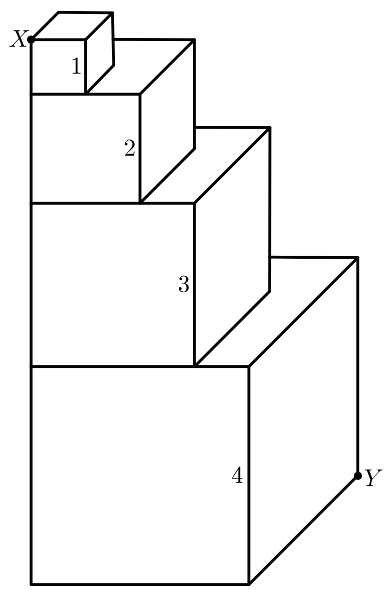
17. Three fair six-sided dice are rolled. What is the probability that the values shown on two of the dice sum to the value shown on the remaining die?

**(A)** $\frac{1}{6}$ **(B)** $\frac{13}{72}$ **(C)** $\frac{7}{36}$ **(D)** $\frac{5}{24}$ **(E)** $\frac{2}{9}$

18. A square in the coordinate plane has vertices whose $y$-coordinates are 0, 1, 4, and 5. What is the area of the square?

**(A)** 16 **(B)** 17 **(C)** 25 **(D)** 26 **(E)** 27

19. Four cubes with edge lengths 1, 2, 3, and 4 are stacked as shown. What is the length of the portion of $\overline{XY}$ contained in the cube with edge length 3?




**(A)** $\frac{3\sqrt{33}}{5}$ **(B)** $2\sqrt{3}$ **(C)** $\frac{2\sqrt{33}}{3}$ **(D)** 4 **(E)** $3\sqrt{2}$

20. The product $(8)(888\ldots8)$, where the second factor has $k$ digits, is an integer whose digits have a sum of 1000. What is $k$?

**(A)** 901 **(B)** 911 **(C)** 919 **(D)** 991 **(E)** 999

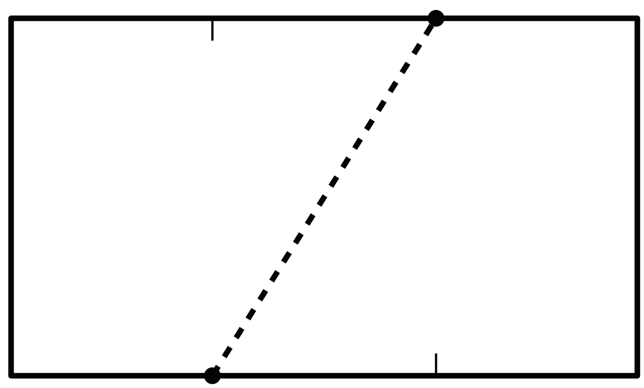
21. Positive integers $a$ and $b$ are such that the graphs of $y = ax + 5$ and $y = 3x + b$ intersect the $x$-axis at the same point. What is the sum of all possible $x$-coordinates of these points of intersection?

**(A)** $-20$ **(B)** $-18$ **(C)** $-15$ **(D)** $-12$ **(E)** $-8$

22. In rectangle $ABCD$, $AB = 20$ and $BC = 10$. Let $E$ be a point on $\overline{CD}$ such that $\angle CBE = 15^\circ$. What is $AE$?

**(A)** $\frac{20\sqrt{3}}{3}$ **(B)** $10\sqrt{3}$ **(C)** 18 **(D)** $11\sqrt{3}$ **(E)** 20

23. A rectangular piece of paper whose length is $\sqrt{3}$ times the width has area $A$. The paper is divided into three equal sections along the opposite lengths, and then a dotted line is drawn from the first divider to the second divider on the opposite side as shown. The paper is then folded flat along this dotted line to create a new shape with area $B$. What is the ratio $B : A$?



**(A)** $1 : 2$ **(B)** $3 : 5$ **(C)** $2 : 3$ **(D)** $3 : 4$ **(E)** $4 : 5$

24. A sequence of natural numbers is constructed by listing the first 4, then skipping one, listing the next 5, skipping 2, listing 6, skipping 3, and, on the $n$th iteration, listing $n+3$ and skipping $n$. The sequence begins $1, 2, 3, 4, 6, 7, 8, 9, 10, 13$. What is the 500,000th number in the sequence?

**(A)** 996,506 **(B)** 996,507 **(C)** 996,508 **(D)** 996,509 **(E)** 996,510

25. The number $5^{867}$ is between $2^{2013}$ and $2^{2014}$. How many pairs of integers $(m, n)$ are there such that $1 \leq m \leq 2012$ and

$$5^n < 2^m < 2^{m+2} < 5^{n+1}?$$

**(A)** 278 **(B)** 279 **(C)** 280 **(D)** 281 **(E)** 282

1. Leah has 13 coins, all of which are pennies and nickels. If she had one more nickel than she has now, then she would have the same number of pennies and nickels. In cents, how much are Leah's coins worth?

**(A)** 33 **(B)** 35 **(C)** 37 **(D)** 39 **(E)** 41

2. What is $\dfrac{2^3+2^3}{2^{-3}+2^{-3}}$?

**(A)** 16 **(B)** 24 **(C)** 32 **(D)** 48 **(E)** 64

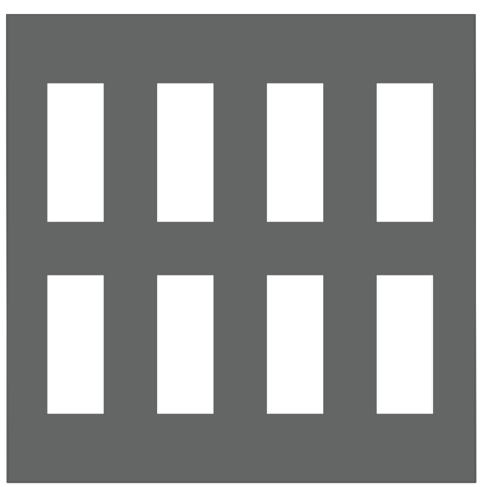
3. Randy drove the first third of his trip on a gravel road, the next 20 miles on pavement, and the remaining one-fifth on a dirt road. In miles, how long was Randy's trip?

**(A)** 30 **(B)** $\dfrac{400}{11}$ **(C)** $\dfrac{75}{2}$ **(D)** 40 **(E)** $\dfrac{300}{7}$

4. Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?

**(A)** $\dfrac{3}{2}$ **(B)** $\dfrac{5}{3}$ **(C)** $\dfrac{7}{4}$ **(D)** 2 **(E)** $\dfrac{13}{4}$

5. Doug constructs a square window using 8 equal-size panes of glass, as shown. The ratio of the height to width for each pane is $5:2$, and the borders around and between the panes are 2 inches wide. In inches, what is the side length of the square window?



**(A)** 26 **(B)** 28 **(C)** 30 **(D)** 32 **(E)** 34

6. Orvin went to the store with just enough money to buy 30 balloons. When he arrived he discovered that the store had a special sale on balloons: buy 1 balloon at the regular price and get a second at $\frac{1}{3}$ off the regular price. What is the greatest number of balloons Orvin could buy?

**(A)** 33 **(B)** 34 **(C)** 36 **(D)** 38 **(E)** 39

7. Suppose $A > B > 0$ and $A$ is $x\%$ greater than $B$. What is $x$?

**(A)** $100\left(\frac{A-B}{B}\right)$ **(B)** $100\left(\frac{A+B}{B}\right)$ **(C)** $100\left(\frac{A+B}{A}\right)$

**(D)** $100\left(\frac{A-B}{A}\right)$ **(E)** $100\left(\frac{A}{B}\right)$

8. A truck travels $\frac{b}{6}$ feet every $t$ seconds. There are 3 feet in a yard. How many yards does the truck travel in 3 minutes?

**(A)** $\frac{b}{1080t}$ **(B)** $\frac{30t}{b}$ **(C)** $\frac{30b}{t}$ **(D)** $\frac{10t}{b}$ **(E)** $\frac{10b}{t}$

9. For real numbers $w$ and $z$,

$$\frac{\frac{1}{w}+\frac{1}{z}}{\frac{1}{w}-\frac{1}{z}} = 2014.$$

What is $\frac{w+z}{w-z}$?

**(A)** $-2014$ **(B)** $\frac{-1}{2014}$ **(C)** $\frac{1}{2014}$ **(D)** 1 **(E)** 2014

10. In the addition shown below $A$, $B$, $C$, and $D$ are distinct digits. How many different values are possible for $D$?

$$\begin{array}{cr} & ABBCB \\ + & BCADA \\ \hline & DBDDD \end{array}$$

**(A)** 2 **(B)** 4 **(C)** 7 **(D)** 8 **(E)** 9

11. For the consumer, a single discount of $n\%$ is more advantageous than any of the following discounts:

(1) two successive 15% discounts

(2) three successive 10% discounts

(3) a 25% discount followed by a 5% discount

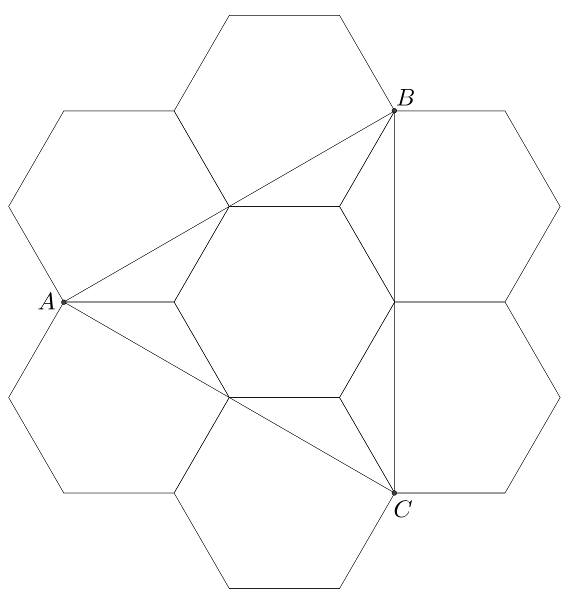
What is the smallest possible positive integer value of $n$?

**(A)** 27 **(B)** 28 **(C)** 29 **(D)** 31 **(E)** 33

12. The largest divisor of 2,014,000,000 is itself. What is its fifth largest divisor?

**(A)** 125,875,000 **(B)** 201,400,000 **(C)** 251,750,000 **(D)** 402,800,000 **(E)** 503,500,000

13. Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of $\triangle ABC$?

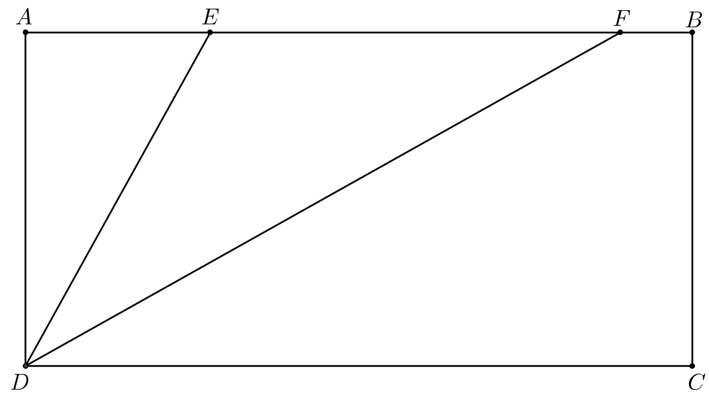



**(A)** $2\sqrt{3}$ **(B)** $3\sqrt{3}$ **(C)** $1+3\sqrt{2}$ **(D)** $2+2\sqrt{3}$ **(E)** $3+2\sqrt{3}$

14. Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, $abc$ miles was displayed on the odometer, where $abc$ is a 3-digit number with $a \geq 1$ and $a+b+c \leq 7$. At the end of the trip, the odometer showed $cba$ miles. What is $a^2+b^2+c^2$?

**(A)** 26 **(B)** 27 **(C)** 36 **(D)** 37 **(E)** 41

15. In rectangle $ABCD$, $DC = 2CB$ and points $E$ and $F$ lie on $\overline{AB}$ so that $\overline{ED}$ and $\overline{FD}$ trisect $\angle ADC$ as shown. What is the ratio of the area of $\triangle DEF$ to the area of rectangle $ABCD$?




**(A)** $\frac{\sqrt{3}}{6}$ **(B)** $\frac{\sqrt{6}}{8}$ **(C)** $\frac{3\sqrt{3}}{16}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{\sqrt{2}}{4}$

16. Four fair six-sided dice are rolled. What is the probability that at least three of the four dice show the same value?

**(A)** $\frac{1}{36}$ **(B)** $\frac{7}{72}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{5}{36}$ **(E)** $\frac{1}{6}$

17. What is the greatest power of 2 that is a factor of $10^{1002} - 4^{501}$?

**(A)** $2^{1002}$ **(B)** $2^{1003}$ **(C)** $2^{1004}$ **(D)** $2^{1005}$ **(E)** $2^{1006}$

18. A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?

**(A)** 24 **(B)** 30 **(C)** 31 **(D)** 33 **(E)** 35

19. Two concentric circles have radii 1 and 2. Two points on the outer circle are chosen independently and uniformly at random. What is the probability that the chord joining the two points intersects the inner circle?

**(A)** $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2-\sqrt{2}}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

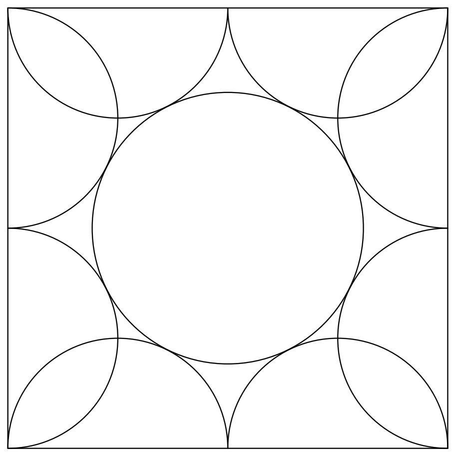
20. For how many integers $x$ is the number $x^4 - 51x^2 + 50$ negative?

**(A)** 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

21. Trapezoid $ABCD$ has parallel sides $\overline{AB}$ of length 33 and $\overline{CD}$ of length 21. The other two sides are of lengths 10 and 14. The angles at $A$ and $B$ are acute. What is the length of the shorter diagonal of $ABCD$?

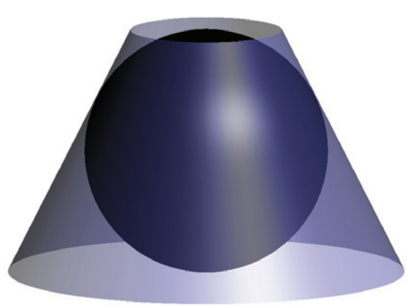
**(A)** $10\sqrt{6}$ **(B)** 25 **(C)** $8\sqrt{10}$ **(D)** $18\sqrt{2}$ **(E)** 26

22. Eight semicircles line the inside of a square with side length 2 as shown. What is the radius of the circle tangent to all of these semicircles?



**(A)** $\frac{1+\sqrt{2}}{4}$ **(B)** $\frac{\sqrt{5}-1}{2}$ **(C)** $\frac{\sqrt{3}+1}{4}$ **(D)** $\frac{2\sqrt{3}}{5}$ **(E)** $\frac{\sqrt{5}}{3}$

23. A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?



**(A)** $\frac{3}{2}$ **(B)** $\frac{1+\sqrt{5}}{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $\frac{3+\sqrt{5}}{2}$

24. The numbers 1, 2, 3, 4, 5 are to be arranged in a circle. An arrangement is *bad* if it is not true that for every $n$ from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to $n$. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

25. In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad $N$, $0 < N < 10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability $1-\frac{N}{10}$. Each jump is independent of the previous jumps. If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape being eaten by the snake?

**(A)** $\frac{32}{79}$ **(B)** $\frac{161}{384}$ **(C)** $\frac{63}{146}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

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