

United Kingdom
Mathematics Trust

Past Paper Collection

CasperYC.Club

Last updated: April 20, 2020



Part I

Junior Challenge Past Paper Collection

Answers

Last updated: April 20, 2020

	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	
1	A	E	B	C	B	C	D	B	B	B	E	E	D	A	B	E	C	A				1
2	D	D	E	B	A	D	E	B	E	E	C	E	B	E	A	D	D	E				2
3	C	A	A	C	C	B	B	D	A	B	E	C	C	D	E	D	A	A				3
4	D	B	D	E	C	E	C	E	D	C	C	C	E	A	C	E	E	E				4
5	A	C	C	B	B	E	E	D	D	B	D	D	A	B	A	C	A	C				5
6	A	B	E	D	D	A	D	B	C	E	D	A	A	B	C	D	D	B				6
7	C	C	C	C	A	D	C	C	A	D	B	D	B	A	A	A	B	A				7
8	E	E	E	A	E	D	A	E	D	A	B	A	A	C	A	B	D	A				8
9	C	B	C	E	E	C	E	A	A	A	C	B	E	D	E	D	C	B				9
10	E	D	B	D	D	D	B	B	E	C	E	E	D	E	D	E	C	E				10
11	E	B	D	A	B	A	E	C	C	C	B	C	D	D	C	B	D	D				11
12	C	A	B	E	C	A	C	A	C	E	B	E	B	B	B	D	A	D				12
13	A	D	A	A	A	D	C	D	B	E	C	D	A	A	C	C	D	E				13
14	B	C	C	D	C	E	D	E	D	A	A	B	D	E	D	B	E	D				14
15	D	C	D	A	D	C	B	E	C	A	E	A	E	C	E	A	D	E				15
16	C	D	C	D	C	B	A	D	E	B	D	B	E	D	B	D	E	C				16
17	B	E	D	E	D	C	B	A	D	D	A	E	C	B	C	B	B	D				17
18	D	D	B	A	A	B	B	B	D	C	D	A	E	B	D	A	B	C				18
19	D	D	B	E	D	A	D	A	C	D	E	D	C	D	B	A	C	B				19
20	B	C	A	B	E	E	A	D	B	A	A	B	B	C	A	B	B	E				20
21	A	C	E	D	A	B	A	E	C	B	B	C	D	C	A	C	D	D				21
22	D	E	D	C	B	D	D	D	E	D	B	E	C	E	E	B	B	A				22
23	C	A	E	B	B	E	B	C	E	E	A	D	D	D	B	E	A	B				23
24	E	B	A	C	C	C	E	C	B	D	D	B	C	B	D	E	B	B				24
25	B	A	D	B	B	B	D	B	A	C	A	C	A	E	D	E	D	D				25



1. How many minutes is it from 23:35 today to 01:15 tomorrow?

- A 100 B 110 C 120 D 130 E 140

2. Which of these is equal to $(0.1 + 0.2 + 0.3 - 0.4) \div 0.5$?

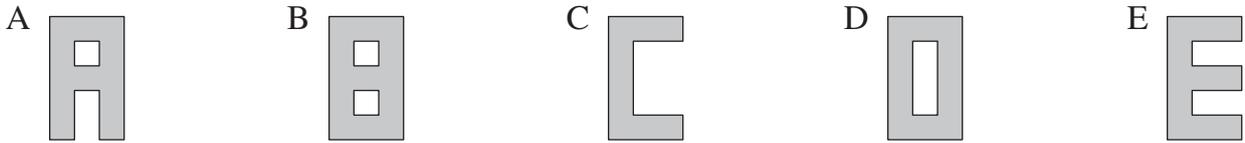
- A 0.01 B 0.02 C 0.04 D 0.1 E 0.4

3. Sam has eaten three-quarters of the grapes.

What is the ratio of the number of grapes that remain to the number Sam has eaten?

- A 1 : 3 B 1 : 4 C 1 : 5 D 1 : 6 E 1 : 7

4. Which of the following five shapes can be cut into four pieces by a single straight cut?



5. On Aoife's 16th birthday, Buster was three times her age. On Aoife's 21st birthday, how old was Buster?

- A 32 B 48 C 53 D 63 E 64

6. Which of these is closest to 7?

- A 7.09 B 6.918 C 7.17 D 6.7 E 7.085

7. The shortest street in the UK, *Ebenezer Place* in Wick, is 2.06 m long. The *Trans-Canada Highway*, one of the world's longest roads, is approximately 7821 km in length.

Approximately, how many times longer than the street is the highway?

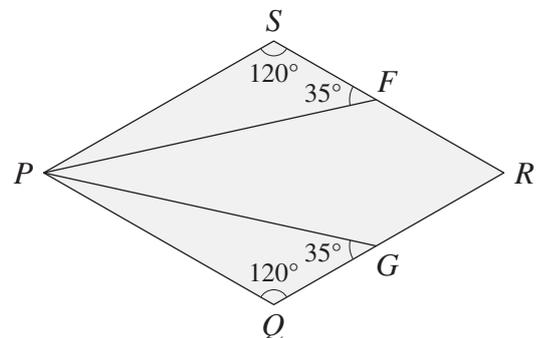
- A 4 000 000 B 400 000 C 40 000 D 4000 E 400

8. The diagram shows a kite $PGRF$ inside rhombus $PQRS$.

Angle $PGQ = 35^\circ$, angle $PFS = 35^\circ$,
angle $PQG = 120^\circ$ and angle $PSF = 120^\circ$.

What is the size of angle FPG ?

- A 10° B 12° C 15° D 18° E 20°



9. What is 50% of 18.3 plus 18.3% of 50?

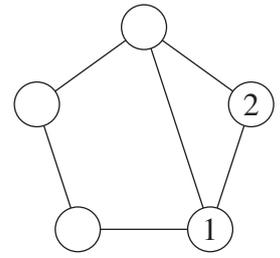
- A 9.15 B 18.3 C 27.15 D 59.15 E 68.3

10. What is the last digit of the smallest positive integer whose digits add to 2019?

- A 1 B 4 C 6 D 8 E 9

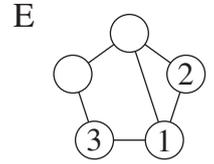
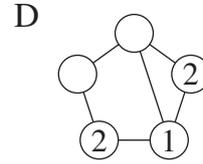
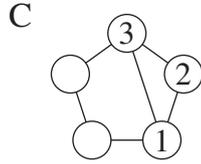
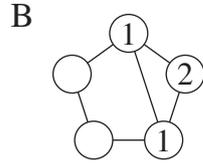
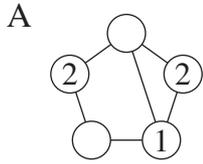


11. Two players X and Y take alternate turns in a game, starting with the diagram alongside.



At each turn, one player writes one of 1, 2 or 3 in an empty circle, so that no two circles connected by an edge contain the same number. A player loses when they cannot go. In each of the five diagrams below it is Y 's turn.

In which of the diagrams can Y 's move ensure that X loses the game?

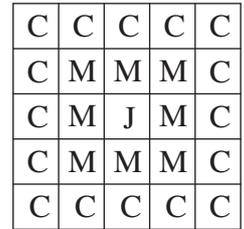


12. Jamal writes down a sequence of six integers. The rule he uses is, “after the first three terms, each term is the sum of the three previous terms.” His sequence is —, —, —, 8, 13, 25.

What is his first term?

- A 0 B 1 C 2 D 3 E 4

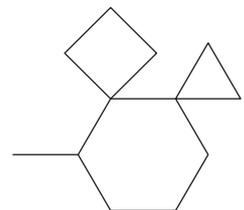
13. In how many different ways can you spell out JMC, starting at the centre, and moving to the next letter in a neighbouring square – horizontally, vertically, or diagonally – each time?



- A 8 B 16 C 24 D 25 E 32

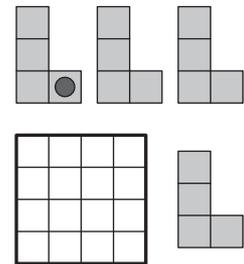
14. Each edge in the diagram has length 1 cm.

What is the length of the longest path that can be followed along the edges, starting at a vertex and without revisiting any vertex?



- A 7 cm B 8 cm C 9 cm D 10 cm E 11 cm

15. All four L-shapes shown in the diagram are to be placed in the 4 by 4 grid so that all sixteen cells are covered and there is no overlap. Each piece can be rotated or reflected before being placed and the black dot is visible from both sides.



How many of the 16 cells of the grid could contain the black dot?

- A 4 B 7 C 8 D 12 E 16

16. Tamsin writes down three two-digit integers. One is square, one is prime and one is triangular. She uses the digits 3, 4, 5, 6, 7 and 8 exactly once each.

Which prime does she write?

- A 37 B 43 C 53 D 73 E 83

17. A rectangle is three times as long as it is high. The area of a square is twelve times the area of the rectangle. What is the ratio of the perimeter of the square to the perimeter of the rectangle?

- A 12 : 1 B 6 : 1 C 4 : 1 D 3 : 1 E 2 : 1



18. What fraction of the integers from 1 to 8000 inclusive are cubes?

- A $\frac{1}{1000}$ B $\frac{1}{800}$ C $\frac{1}{400}$ D $\frac{1}{200}$ E $\frac{1}{100}$

19. Each row, each column and each of the bold 2 by 3 rectangles in the grid has to contain each of the numbers 1, 2, 3, 4, 5 and 6 (one number in each cell).

				<i>x</i>	5
				6	
		1	2		
		3	4		
		4		3	
2					1

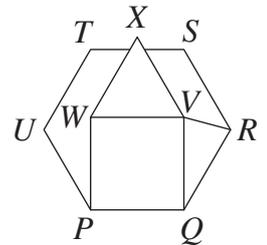
What number should go in the cell marked *x*?

- A 1 B 2 C 3 D 4 E 6

20. Emily writes down the largest two-digit prime such that each of its digits is prime.
 Krish writes down the smallest two-digit prime such that each of its digits is prime.
 Kirsten subtracts Krish's number from Emily's number.
 What answer does Kirsten obtain?

- A 14 B 20 C 36 D 45 E 50

21. The diagram shows a regular hexagon *PQRSTU*, a square *PQVW* and an equilateral triangle *VXW*.



What is the size of angle *XVR*?

- A 120° B 125° C 130° D 135° E 140°

22. In the multiplication shown alongside, *T*, *R*, *A* and *P* are all different digits.

$$\begin{array}{r} T R A P \\ \times \quad 9 \\ \hline P A R T \end{array}$$

What is the value of *R*?

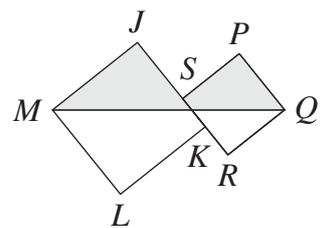
- A 0 B 1 C 5 D 8 E 9

23. The diagram shows two squares *JKLM* and *PQRS*.

The length of *JK* is 6 cm and that of *PQ* is 4 cm.

The vertex *K* is the midpoint of side *RS*.

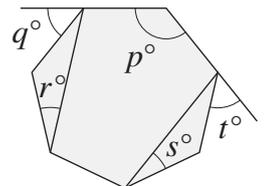
What is the area of the shaded region?



- A 22 cm^2 B 24 cm^2 C 26 cm^2 D 28 cm^2
 E 30 cm^2

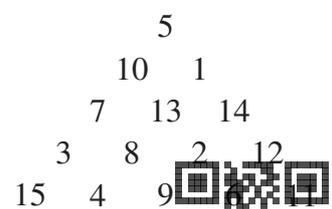
24. The diagram shows a regular heptagon.

Which of these expressions is equal to $p + q + r + s + t$?



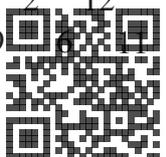
- A $180 + q$ B $180 + 2q$ C $360 - q$ D 360
 E $360 + q$

25. The diagram shows the first fifteen positive integers arranged in a 'triangle'.
 These numbers are to be rearranged so that the five integers along each 'edge' of the triangle have the same sum, unlike the example shown.



When this is done, what is the greatest possible such sum?

- A 38 B 42 C 48 D 52 E 54



1. What is the value of $(222 + 22) \div 2$?
 A 111 B 112 C 122 D 133 E 233
2. A train carriage has 80 seats. On my journey I noticed that all the seats in my carriage were taken and 7 people were standing.

At Banbury, 9 people left the carriage, 28 people entered it and all seats were taken.

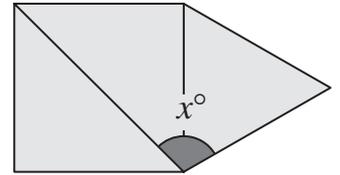
How many people now had no seat?

- A 0 B 7 C 16 D 26 E 35

3. The diagram shows an equilateral triangle, a square, and one diagonal of the square.

What is the value of x ?

- A 105 B 110 C 115 D 120 E 135



4. The perimeter of the regular decagon P is 8 times the perimeter of the regular octagon Q .
 Each edge of the regular octagon Q is 10 cm long.

How long is each edge of the regular decagon P ?

- A 8 cm B 10 cm C 40 cm D 60 cm E 64 cm

5. My train left Southampton at 06:15 and arrived in Birmingham at 08:48 later that morning.
 How many minutes did the journey take?

- A 153 B 193 C 233 D 1463 E 1501

6. The diagram shows a partially completed magic square, in which all rows, all columns and both main diagonals have the same total.

What is the value of $x + y$?

- A 10 B 11 C 12 D 13 E 14

4		
	7	y
6	5	x

7. How many integers are greater than $20 + 18$ and also less than 20×18 ?

- A 320 B 321 C 322 D 323 E 324

8. Gill scored a goal half way through the second quarter of a 'teachers versus pupils' netball match. At that point, what fraction of the whole match remained to be played?

- A $\frac{1}{4}$ B $\frac{3}{8}$ C $\frac{1}{2}$ D $\frac{5}{8}$ E $\frac{3}{4}$

9. The approximate cost of restoring the Flying Scotsman was £4 million. This was about 500 times the cost of building the steam engine in 1923.

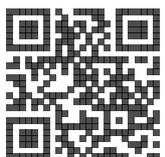
Roughly what did the engine cost to build?

- A £800 B £2000 C £8000 D £20 000 E £80 000

10. Adding four of the five fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$ and $\frac{1}{18}$ gives a total of 1.

Which of the fractions is not used?

- A $\frac{1}{2}$ B $\frac{1}{3}$ C $\frac{1}{6}$ D $\frac{1}{9}$ E $\frac{1}{18}$

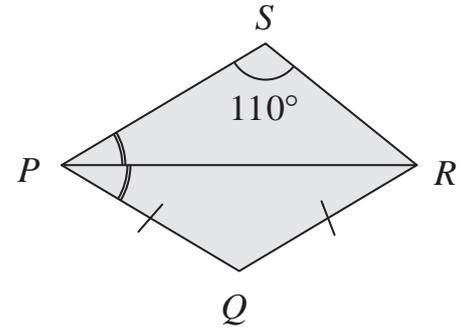


11. How many digits are there in the correct answer to the calculation $123\ 123\ 123\ 123 \div 123$?
 A 4 B 6 C 8 D 10 E 12

12. The diagram shows a quadrilateral $PQRS$ in which PQ and QR have the same length. Also PR bisects $\angle SPQ$, the ratio of $\angle SPR$ to $\angle PRS$ is $2 : 3$ and $\angle PSR = 110^\circ$.

How large is angle PQR ?

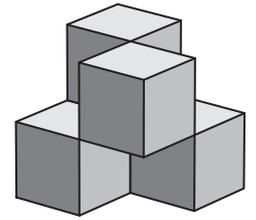
- A 124° B 120° C 110° D 90° E 28°



13. The diagram shows a shape made from four $3\text{ cm} \times 3\text{ cm} \times 3\text{ cm}$ wooden cubes joined by their edges.

What, in cm^2 , is the surface area of the shape?

- A 162 B 180 C 198 D 216 E 234



14. Billy has three times as many llamas as lambs.

Milly has twice as many lambs as llamas.

They have 17 animals in total.

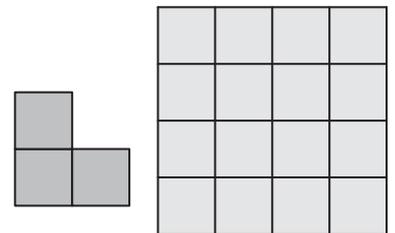
How many of the animals are llamas?

- A 5 B 6 C 7 D 8 E 9

15. Beatrix places copies of the L-shape shown on a 4×4 board so that each L-shape covers exactly three cells of the board. She is allowed to turn around or turn over an L-shape.

What is the largest number of L-shapes she can place on the board without overlaps?

- A 2 B 3 C 4 D 5 E 6



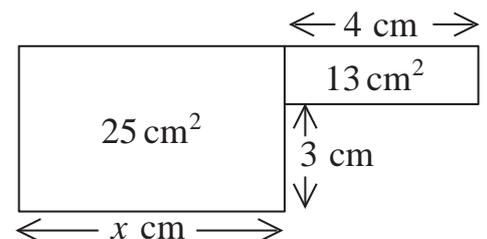
16. How many pairs of digits (p, q) are there so that the five-digit integer ' $p869q$ ' is a multiple of 15?

- A 2 B 3 C 4 D 5 E 6

17. The areas of the two rectangles in the diagram are 25 cm^2 and 13 cm^2 as indicated.

What is the value of x ?

- A 3 B 4 C 5 D 6 E 7



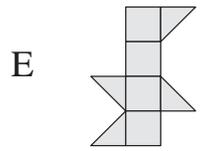
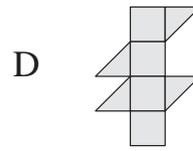
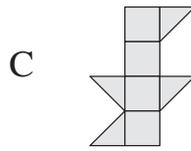
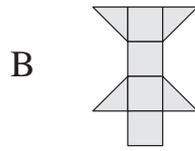
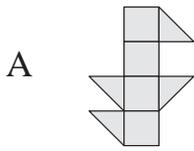
18. Between them, the two five-digit integers M and N contain all ten digits from 0 to 9.

What is the least possible difference between M and N ?

- A 123 B 247 C 427 D 472 E 742



19. Which one of these could be folded to make a cube?



20. A drawer contains ten identical yellow socks, eight identical blue socks and four identical pink socks.

Amrita picks socks from the drawer without looking.

What is the smallest number of socks she must pick to be sure that she has at least two pairs of matching socks?

- A 5 B 6 C 8 D 11 E 13

21. There are _____ vowels in this short sentence.

Which of the following options should replace "_____" to make the sentence in the box true?

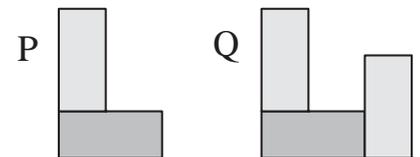
- A twelve B thirteen C fourteen D fifteen E sixteen
22. In the triangles PQR and STU , $\angle RPQ = 2 \times \angle UST$, $\angle PRQ = 2 \times \angle SUT$ and $\angle RQP = \frac{1}{5} \times \angle UTS$.
- How large is $\angle UTS$?
- A 90° B 100° C 120° D 150° E more information needed

23. Ali wants to fill the empty squares so that the number in each square after the fourth from the left is the sum of the numbers in the four squares to its left.



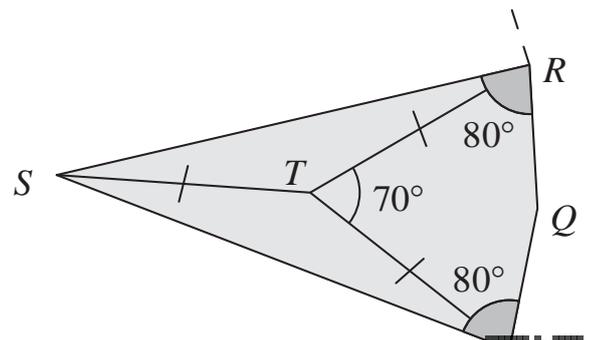
What number should Ali write in the final square?

- A 16 B 8 C 4 D 2 E 1
24. The shapes P and Q are formed from two and three identical rectangles, respectively. Their perimeters are 58 cm and 85 cm respectively.



What is the perimeter of one of the rectangles?

- A 30 cm B 31 cm C 32 cm D 33 cm E 34 cm
25. In the diagram PQ and QR are sides of a regular n -sided polygon, $\angle SPQ = \angle SRQ = 80^\circ$, $\angle PTR = 70^\circ$ and $PT = ST = RT$.



What is the value of n ?

- A 15 B 18 C 20 D 24 E 30



1. Which of the following calculations gives the largest answer?

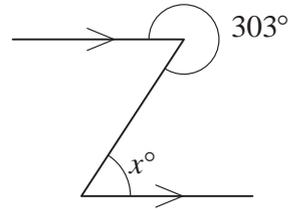
A $2 - 1$ B $2 \div 1$ C 2×1 D 1×2 E $2 + 1$

2. Nadiya is baking a cake. The recipe says that her cake should be baked in the oven for 1 hour and 35 minutes. She puts the cake in the oven at 11:40 am. At what time should she take the cake out of the oven?

A 12:15 pm B 12:40 pm C 1:05 pm D 1:15 pm E 2:15 pm

3. What is the value of x ?

A 43 B 47 C 53 D 57 E 67



4. A download is 95% complete. What fraction is yet to be downloaded?

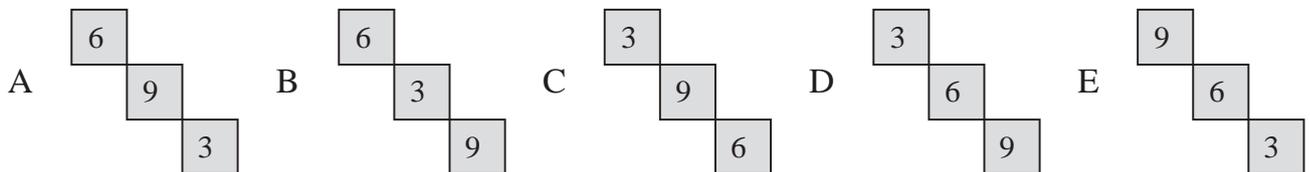
A $\frac{1}{2}$ B $\frac{1}{5}$ C $\frac{1}{9}$ D $\frac{1}{10}$ E $\frac{1}{20}$

5. What is the value of $201 \times 7 - 7 \times 102$?

A 142 800 B 793 C 693 D 607 E 0

6. In a magic square, the numbers in each row, each column and the two main diagonals have the same total. This magic square uses the integers 2 to 10. Which of the following are the missing cells?

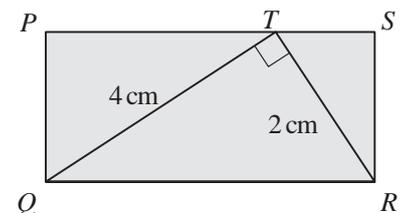
	10	5
8		4
7	2	



7. If you work out the values of the following expressions and then place them in increasing numerical order, which comes in the middle?

A $\frac{2}{3} + \frac{4}{5}$ B $\frac{2}{3} \times \frac{4}{5}$ C $\frac{3}{2} + \frac{5}{4}$ D $\frac{2}{3} \div \frac{4}{5}$ E $\frac{3}{2} \times \frac{5}{4}$

8. The diagram shows a rectangle $PQRS$ and T is a point on PS such that QT is perpendicular to RT . The length of QT is 4 cm. The length of RT is 2 cm.



What is the area of the rectangle $PQRS$?

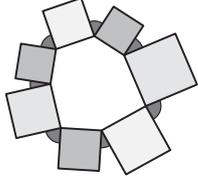
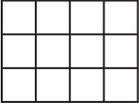
A 6 cm^2 B 8 cm^2 C 10 cm^2 D 12 cm^2 E 16 cm^2

9. In William Shakespeare's play *As You Like It*, Rosalind speaks to Orlando about "He that will divide a minute into a thousand parts".

Which of the following is equal to the number of seconds in one thousandth of one minute?

A 0.24 B 0.6 C 0.024 D 0.06 E 0.006



10. Which of the following integers is not a multiple of 45?
 A 765 B 675 C 585 D 495 E 305
11. Seven squares are drawn on the sides of a heptagon so that they are outside the heptagon, as shown in the diagram.
 What is the sum of the seven marked angles?
 A 315° B 360° C 420° D 450° E 630°
- 
12. Last year, at the school where Gill teaches Mathematics, 315 out of the 600 pupils were girls. This year, the number of pupils in the school has increased to 640. The proportion of girls is the same as it was last year.
 How many girls are there at the school this year?
 A 339 B 338 C 337 D 336 E 335
13. Consider the following three statements.
 (i) Doubling a positive number always makes it larger.
 (ii) Squaring a positive number always makes it larger.
 (iii) Taking the positive square root of a positive number always makes it smaller.
 Which statements are true?
 A All three B None C Only (i) D (i) and (ii) E (ii) and (iii)
14. Mathias is given a grid of twelve small squares. He is asked to shade grey exactly four of the small squares so that his grid has two lines of reflection symmetry.
 How many different grids could he produce?
 A 2 B 3 C 4 D 5 E 6
- 
15. What is the remainder when the square of 49 is divided by the square root of 49?
 A 0 B 2 C 3 D 4 E 7

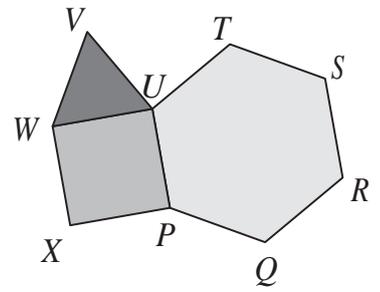
16. In New Threeland there are three types of coins: the 2p; the 5p; and one other. The smallest number of coins needed to make 13p is three. The smallest number of coins needed to make 19p is three. What is the value of the third type of coin?
 A 4p B 6p C 7p D 9p E 12p
17. I add up all even numbers between 1 and 101. Then from my total I subtract all odd numbers between 0 and 100.
 What is the result?
 A 0 B 50 C 100 D 255 E 2525
18. What is the sum of the digits in the completed crossnumber?

ACROSS	DOWN	
1. A cube	2. A square	
3. A power of 11		

- A 25 B 29 C 32 D 34 E 35



19. The diagram shows a regular hexagon $PQRSTU$, a square $PUWX$ and an equilateral triangle UVW .



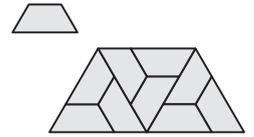
What is the angle TVU ?

- A 45° B 42° C 39° D 36° E 33°

20. The range of a list of integers is 20, and the median is 17.
What is the smallest possible number of integers in the list?

- A 1 B 2 C 3 D 4 E 5

21. The small trapezium on the right has three equal sides and angles of 60° and 120° . Nine copies of this trapezium can be placed together to make a larger version of it, as shown.



The larger trapezium has perimeter 18 cm.

What is the perimeter of the smaller trapezium?

- A 2 cm B 4 cm C 6 cm D 8 cm E 9 cm

22. In the window of Bradley's Bicycle Bazaar there are some unicycles, some bicycles and some tricycles. Laura sees that there are seven saddles in total, thirteen wheels in total and more bicycles than tricycles.

How many unicycles are in the window?

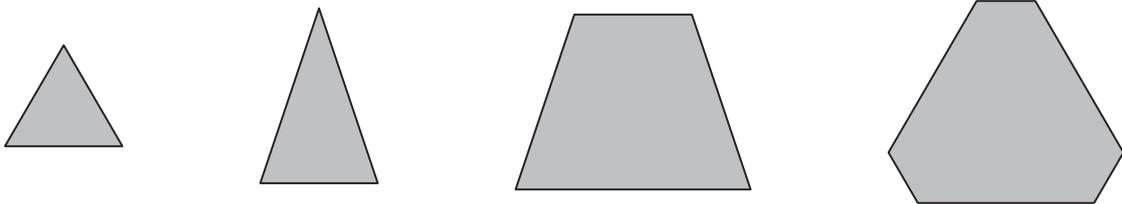
- A 1 B 2 C 3 D 4 E 5

23. The positive integers from 1 to 150 inclusive are placed in a 10 by 15 grid so that each cell contains exactly one integer. Then the multiples of 3 are given a red mark, the multiples of 5 are given a blue mark, and the multiples of 7 are given a green mark.

How many cells have more than 1 mark?

- A 10 B 12 C 15 D 18 E 19

24. A large solid cube is cut into two pieces by a single plane cut. How many of the following four shapes could be the shape of the cross-section formed by the cut?

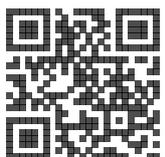


- A 0 B 1 C 2 D 3 E 4

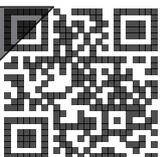
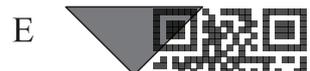
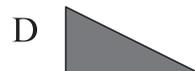
25. The distance between Exeter and London is 175 miles. Sam left Exeter at 10:00 on Tuesday for London. Morgan left London for Exeter at 13:00 the same day. They travelled on the same road. Up to the time when they met, Sam's average speed was 25 miles per hour, and Morgan's average speed was 35 miles an hour.

At what time did Sam and Morgan meet?

- A 17:00 B 15:55 C 15:30 D 15:00 E 14:40



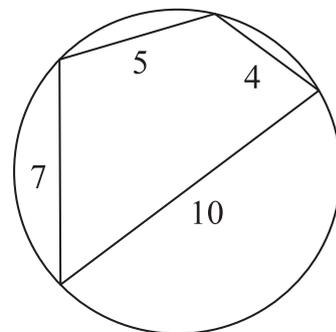
1. Which of the following is closest to zero?
 A $6 + 5 + 4$ B $6 + 5 - 4$ C $6 + 5 \times 4$ D $6 - 5 \times 4$ E $6 \times 5 \div 4$
2. What number is twenty-one less than sixty thousand?
 A 59 979 B 59 981 C 57 900 D 40 001 E 39 000
3. One lap of a standard running track is 400 m.
 How many laps does each athlete run in a 5000 m race?
 A 4 B 5 C 8 D 10 E $12\frac{1}{2}$
4. In January 1859, an eight-year-old boy dropped a newly-hatched eel into a well in Sweden (apparently in order to keep the water free of insects). The eel, named Åle, finally died in August 2014.
 How many years old was Åle when it died?
 A 135 B 145 C 155 D 165 E 175
5. What is the value of $\frac{1}{25} + 0.25$?
 A 0.29 B 0.3 C 0.35 D 0.50 E 0.65
6. Gill is now 28 years old and is a teacher of Mathematics at a school which has 600 pupils. There are 30 more girls than boys at the school.
 How many girls are at Gill's school?
 A 270 B 300 C 315 D 330 E 345
7. A distance of 8 km is approximately 5 miles.
 Which of the following is closest to 1.2 km?
 A 0.75 miles B 1 mile C 1.2 miles D 1.6 miles E 1.9 miles
8. What is the value of $\frac{2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20}{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10}$?
 A 2 B 10 C 20 D 40 E 1024
9. One of the three symbols $+$, $-$, \times is inserted somewhere between the digits of 2016 to give a new number. For example, $20 - 16$ gives 4.
 How many of the following four numbers can be obtained in this way?
 36 195 207 320
 A 0 B 1 C 2 D 3 E 4
10. A square is folded exactly in half and then in half again.
 Which of the following could not be the resulting shape?



20. A cyclic quadrilateral has all four vertices on the circumference of a circle. Brahmagupta (598–670AD) gave the following formula for the area, A , of a cyclic quadrilateral whose edges have lengths a, b, c, d : $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$, where s is half of the perimeter of the quadrilateral.

What is the area of the cyclic quadrilateral with sides of length 4 cm, 5 cm, 7 cm and 10 cm?

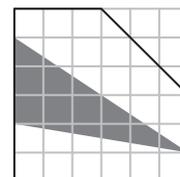
- A 6 cm^2 B 13 cm^2 C 26 cm^2 D 30 cm^2 E 36 cm^2



21. The diagram shows a pentagon drawn on a square grid. All vertices of the pentagon and triangle are grid points.

What fraction of the area of the pentagon is shaded?

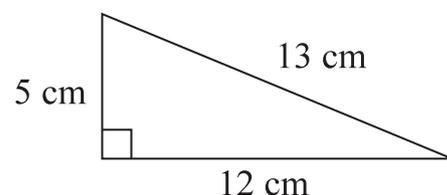
- A $\frac{2}{7}$ B $\frac{1}{3}$ C $\frac{2}{5}$ D $\frac{1}{4}$ E $\frac{2}{9}$



22. Four copies of the triangle shown are joined together, without gaps or overlaps, to make a parallelogram.

What is the largest possible perimeter of the parallelogram?

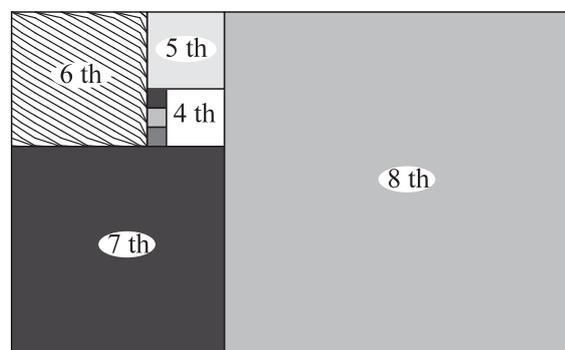
- A 46 cm B 52 cm C 58 cm D 62 cm E 76 cm



23. The diagram shows the first few squares of a 'spiral' sequence of squares. All but the first three squares have been numbered. After the first six squares, the sequence is continued by placing the next square alongside three existing squares – the largest existing square and two others.

The three smallest squares have sides of length 1. What is the side length of the 12th square?

- A 153 B 123 C 83 D 53 E 13



24. Part of a wall is to be decorated with a row of four square tiles. Three different colours of tiles are available and there are at least two tiles of each colour available. Tiles of all three colours must be used.

In how many ways can the row of four tiles be chosen?

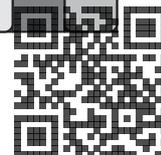
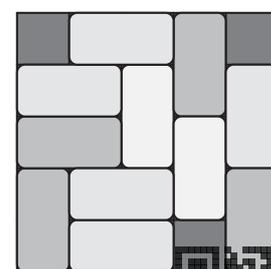
- A 12 B 18 C 24 D 36 E 48



25. Beatrix places dominoes on a 5×5 board, either horizontally or vertically, so that each domino covers two small squares. She stops when she cannot place another domino, as in the example shown in the diagram.

When Beatrix stops, what is the largest possible number of squares that may still be uncovered?

- A 4 B 5 C 6 D 7 E 8



Part II

Intermediate Challenge Past Paper Collection

Answers

Last updated: April 20, 2020

	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	
1	E	D	E	C	D	E	B	D	B	B	D	A	A	B	D	B	B	A			1
2	D	E	D	B	C	C	D	A	A	D	B	A	E	D	B	B	E	E			2
3	A	E	A	D	E	B	C	E	E	E	A	A	E	B	A	C	D	B			3
4	C	C	B	E	E	B	A	A	E	C	D	B	C	E	D	D	B	D			4
5	E	B	C	B	D	D	D	E	C	D	B	D	B	A	B	B	D	D			5
6	D	B	D	A	A	B	A	C	B	A	B	D	D	C	D	A	B	C			6
7	C	D	B	D	B	D	C	D	A	C	E	C	E	C	C	B	B	B			7
8	D	D	A	C	B	A	A	B	E	A	C	B	B	E	D	A	E	E			8
9	E	E	D	D	B	D	B	C	A	B	D	D	B	C	B	E	C	E			9
10	C	E	C	B	C	C	D	C	E	D	A	C	C	E	A	E	B	B			10
11	B	A	C	C	A	E	E	B	E	C	B	E	D	D	E	B	C	E			11
12	B	A	B	D	A	C	D	D	D	B	D	B	D	D	D	E	C	B			12
13	C	C	D	E	E	D	A	B	D	D	B	B	A	B	A	C	A	C			13
14	A	B	C	D	A	B	E	C	E	A	D	D	C	D	B	C	C	A			14
15	E	D	A	A	D	A	B	E	A	E	C	C	C	A	C	E	A	A			15
16	C	C	B	B	D	D	B	E	C	B	C	B	D	B	D	C	A	E			16
17	A	B	C	E	C	D	D	A	C	E	A	A	D	B	B	D	D	E			17
18	E	E	E	E	C	C	B	A	D	C	D	C	A	C	E	A	B	C			18
19	B	D	C	B	E	E	E	B	B	A	A	B	A	B	E	D	E	B			19
20	D	C	D	D	C	A	D	D	D	D	E	E	C	D	D	E	B	D			20
21	D	D	E	A	B	B	E	E	B	A	E	E	D	A	A	D	D	B			21
22	C	A	A	B	A	A	D	E	B	E	D	C	B	E	C	E	E	C			22
23	A	C	E	B	C	E	C	B	A	C	E	E	B	B	D	A	A	A			23
24	B	A	A	C	D	C	C	B	C	D	B	D	E	C	C	B	C	A			24
25	A	B	B	D	E	E	E	D	D	B	D	E	D	D	A	D	D	A			25

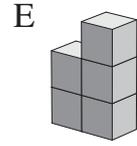
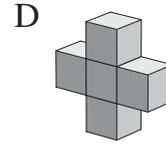
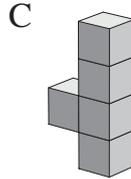
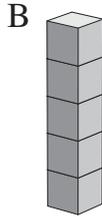
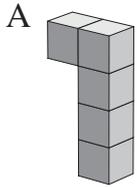


1. What is the value of 2019 tenths?

- A 2019 B 201.9 C 20.19 D 2.019 E 0.2019

2. Each of the five shapes shown below is made from five unit cubes.

Which has the smallest surface area?



3. There are 120 000 red squirrels living in Scotland. This represents 75% of their total UK population.

How many more red squirrels live in Scotland than live in the remainder of the UK?

- A 30 000 B 40 000 C 60 000 D 80 000 E 90 000

4. A 24-hour digital clock shows the time in hours and minutes.

How many times in one day will it display all four digits 2, 0, 1 and 9 in some order?



- A 6 B 10 C 12 D 18 E 24

5. The answers to the three calculations below are to be written in descending order.

$$X \quad 0.6 \times 0.5 + 0.4$$

$$Y \quad 0.6 \times 0.5 \div 0.4$$

$$Z \quad 0.6 \times 0.5 \times 0.4$$

What is the correct order?

- A YZX B XYZ C XZY D YXZ E ZYX

6. The diagram shows part of a tessellation of the plane by a quadrilateral.

Khelen wants to colour each quadrilateral in the pattern so that no two quadrilaterals that meet (even at a point) have the same colour.

What is the smallest number of colours he needs?



- A 3 B 4 C 5 D 6 E 7

7. How many positive cubes less than 5000 end in the digit 5?

- A 1 B 2 C 3 D 4 E 5

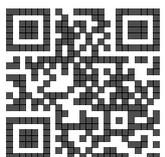
8. Three consecutive positive integers less than 20 are, in ascending order, prime, even and triangular.

What is the product of these three integers?

- A 6 B 60 C 990 D 1786 E 2730

9. What is the value of $(7 - 6 \times (-5)) - 4 \times (-3) \div (-2)$?

- A -17 B -11 C 31 D 37 E 43



10. A recent report about the amount of plastic created in the last 65 years stated that the 8.3 billion tonnes produced is as heavy as 25 000 Empire State Buildings in New York or a billion elephants. On that basis, how many elephants have the same total weight as the Empire State Building?

A 4000 B 40 000 C 400 000 D 4 000 000 E 40 000 000

11. Which of the following is equal to $\frac{3^9}{9^3}$?

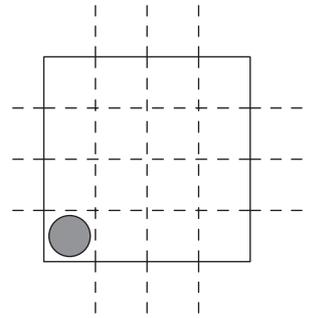
A 3 B 9 C 27 D 81 E 243

12. The game of *Rorrim2* is played on a 4×4 board, starting with a counter in one corner, as shown.

At each turn, the player moves the counter to a cell that is the reflection of its current cell in one of the six dashed lines.

How many cells could the counter occupy after precisely three turns?

A 4 B 6 C 8 D 12 E 16



13. Megan writes down a list of five numbers. The mean of her first three numbers is -3 . The mean of her first four numbers is 4. The mean of her first five numbers is -5 .

What is the difference between her fourth number and her fifth number?

A 66 B 55 C 44 D 33 E 22

14. There are four people, some of whom always tell the truth. The others always lie.

The first person said, "An odd number of us always tell the truth".

The second person said, "An even number of us always tell the truth".

The third person said, "A prime number of us always tell the truth".

The fourth person said, "A square number of us always tell the truth".

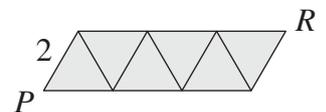
How many of these four people were telling the truth?

A 0 B 1 C 2 D 3 E 4

15. The diagram shows six congruent equilateral triangles, of side-length 2, placed together to form a parallelogram.

What is the length of PR ?

A $2\sqrt{13}$ B 7 C $6\sqrt{3}$ D 9 E $7\sqrt{3}$



16. Two numbers x and y have a product which is equal to their sum.

Which of these expressions gives x in terms of y ?

A $\frac{y}{y-1}$ B $\frac{y}{y+1}$ C $\frac{y+1}{y}$ D $\frac{y-1}{y}$ E $\frac{y^2}{y+1}$

17. Which of these is equal to $0.\dot{8} + 0.0\dot{7}$?

A $0.8\dot{7}$ B 0.88 C $0.9\dot{5}$ D $0.9\dot{6}$ E $0.9\dot{8}$



18. Two numbers x and y are such that $x + y = \frac{2}{3}$ and $\frac{x}{y} = \frac{2}{3}$.

What is the value of $x - y$?

- A $-\frac{2}{3}$ B $-\frac{2}{15}$ C $\frac{2}{25}$ D $\frac{2}{5}$ E $\frac{2}{3}$

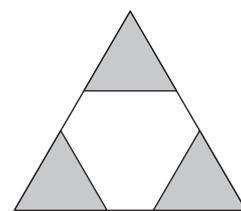
19. Which of these expressions has the largest value?

- A $\frac{1}{2}$ B $\frac{1}{3} + \frac{1}{4}$ C $\frac{1}{4} + \frac{1}{5} + \frac{1}{6}$ D $\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$
 E $\frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10}$

20. Three equilateral triangles with sides of length 1 are shown shaded in a larger equilateral triangle. The total shaded area is half the area of the larger triangle.

What is the side-length of the larger equilateral triangle?

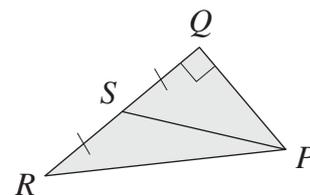
- A $\sqrt{5}$ B $\sqrt{6}$ C $\frac{5}{2}$ D $\frac{3\sqrt{3}}{2}$ E $1 + \sqrt{3}$



21. The diagram shows a right-angled triangle PQR . The point S is the midpoint of the side QR and $\tan \angle QPR = \frac{3}{2}$.

What is the value of $\sin \angle QPS$?

- A $\frac{1}{\sqrt{3}}$ B $\frac{1}{\sqrt{2}}$ C $\frac{1}{2}$ D $\frac{3}{5}$ E $\frac{4}{5}$



22. Four of the following six-digit integers are always divisible by 7, regardless of the values of the digits P and Q .

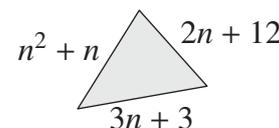
Which of the following is not necessarily a multiple of 7?

- A 'PQQPQQ' B 'PQPQPQ' C 'QPQPQP' D 'PPP PPP' E 'PPP QQQ'

23. The diagram shows a triangle with sides $n^2 + n$, $2n + 12$ and $3n + 3$.

What is the sum of all the values of n for which the triangle is isosceles?

- A 7 B 9 C 12 D 13 E 16



24. When 5655 is divided by a two-digit positive integer N , the remainder is 11. When 5879 is divided by the same positive integer N , the remainder is 14.

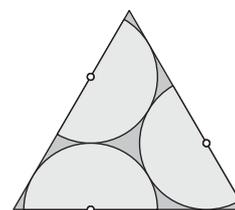
What is the sum of the digits of N ?

- A 6 B 7 C 8 D 9 E 10

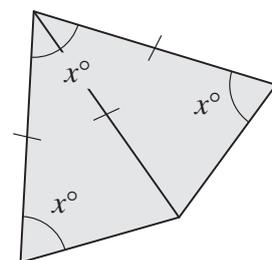
25. The diagram shows three touching semicircles with radius 1 inside an equilateral triangle, which each semicircle also touches. The diameter of each semicircle lies along a side of the triangle.

What is the length of each side of the equilateral triangle?

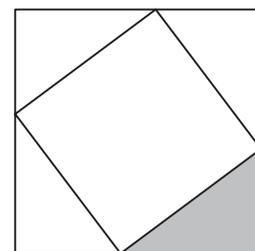
- A 3 B $\frac{13}{4}$ C $\frac{3}{2}\sqrt{3}$ D $2\sqrt{3}$ E 4



1. Which of these is the sum of the cubes of two consecutive integers?
 A 4 B 9 C 16 D 25 E 36
2. How many of these four integers are prime?
 1 11 111 1111
 A 0 B 1 C 2 D 3 E 4
3. In September 2016 a polymer £5 note was introduced. The Bank of England issued 440 million of them.
 What is the total face value of all these notes?
 A £220 000 000 B £440 000 000 C £2 200 000 000
 D £4 400 000 000 E £22 000 000 000
4. A kite is made by joining two congruent isosceles triangles, as shown.
 What is the value of x ?
 A 36 B 54 C 60 D 72 E 80



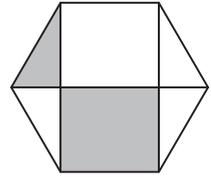
5. The adult human body has 206 bones. Each foot has 26 bones.
 Approximately what fraction of the number of bones in the human body is found in one foot?
 A $\frac{1}{6}$ B $\frac{1}{8}$ C $\frac{1}{10}$ D $\frac{1}{12}$ E $\frac{1}{20}$
6. In 2014, in Boston, Massachusetts, Eli Bishop set a world record for the greatest number of claps per minute. He achieved 1020 claps in one minute.
 How many claps is that per second?
 A 17 B 16.5 C 16 D 15.5 E 15
7. How many two-digit squares have the property that the product of their digits is also a square?
 A 0 B 1 C 2 D 3 E 4
8. The diagram shows a square of perimeter 20 cm inscribed inside a square of perimeter 28 cm.
 What is the area of the shaded triangle?
 A 6 cm^2 B 7 cm^2 C 8 cm^2 D 9 cm^2 E 10 cm^2



9. Which integer n satisfies $\frac{3}{10} < \frac{n}{20} < \frac{2}{5}$?
 A 3 B 4 C 5 D 6 E 7
10. Which of these integers cannot be expressed as the difference of two squares?
 A 5 B 7 C 8 D 9 E 10



11. The diagram shows a regular hexagon which has been divided into six regions by three of its diagonals. Two of these regions have been shaded. The total shaded area is 20 cm^2 .



What is the area of the hexagon?

A 40 cm^2 B 48 cm^2 C 52 cm^2 D 54 cm^2 E 60 cm^2

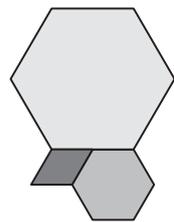
12. Someone has switched the numbers around on Harry's calculator! The numbers should be in the positions shown in the left-hand diagram, but have been switched to the positions in the right-hand diagram.

7	8	9	9	8	7
4	5	6	6	5	4
1	2	3	3	2	1

Which of the following calculations will *not* give the correct answer when Harry uses his calculator?

A 79×97 B 78×98 C 147×369 D 123×321 E 159×951

13. The diagram shows a rhombus and two sizes of regular hexagon. What is the ratio of the area of the smaller hexagon to the area of the larger hexagon?



A 1:2 B 1:3 C 1:4 D 1:8 E 1:9

14. Which of these is equal to $\frac{10}{9} + \frac{9}{10}$?

A 1 B 2 C 2.0 $\dot{1}$ D 2. $\dot{1}$ E 2. $\dot{2}$

15. How many of these four shapes could be the shape of the region where two triangles overlap?

equilateral triangle

square

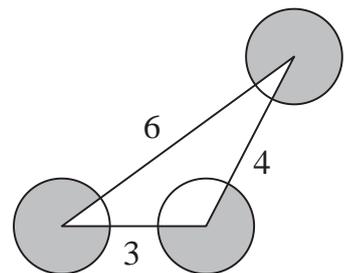
regular pentagon

regular hexagon

A 0 B 1 C 2 D 3 E 4

16. The diagram shows a triangle with edges of length 3, 4 and 6. A circle of radius 1 is drawn at each vertex of the triangle. What is the total shaded area?

A 2π B $\frac{9\pi}{4}$ C $\frac{5\pi}{2}$ D $\frac{11\pi}{4}$ E 3π

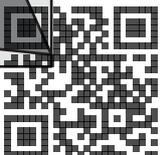
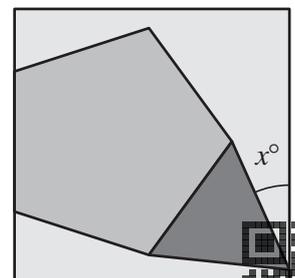


17. How many three-digit numbers are increased by 99 when their digits are reversed?

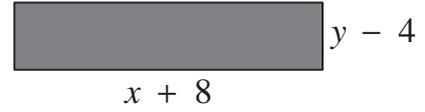
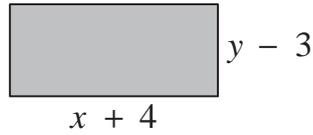
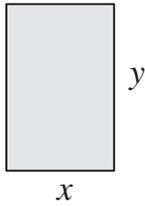
A 4 B 8 C 10 D 80 E 90

18. The diagram shows a regular pentagon and an equilateral triangle placed inside a square. What is the value of x ?

A 24 B 26 C 28 D 30 E 32



19. The three rectangles shown below all have the same area.



What is the value of $x + y$?

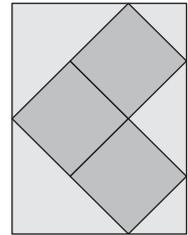
- A 4 B 6 C 8 D 10 E 12
20. A particular integer is the smallest multiple of 72, each of whose digits is either 0 or 1. How many digits does this integer have?

A 4 B 6 C 8 D 10 E 12

21. For certain values of x , the list $x, x + 6$ and x^2 contains just two different numbers. How many such values of x are there?

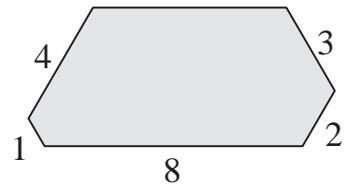
A 1 B 2 C 3 D 4 E 5

22. Three squares, with side-lengths 2, are placed together edge-to-edge to make an L-shape. The L-shape is placed inside a rectangle so that all five vertices of the L-shape lie on the rectangle, one of them at the midpoint of an edge, as shown.



What is the area of the rectangle?

- A 16 B 18 C 20 D 22 E 24
23. The diagram shows a hexagon. All the interior angles of the hexagon are 120° . The lengths of some of the sides are indicated.

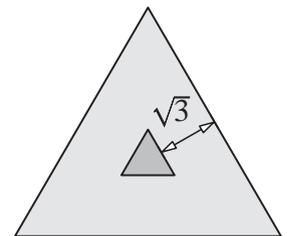


What is the area of the hexagon?

- A $20\sqrt{3}$ B $21\sqrt{3}$ C $22\sqrt{3}$ D $23\sqrt{3}$ E $24\sqrt{3}$
24. A list of 5 positive integers has mean 5, mode 5, median 5 and range 5. How many such lists of 5 positive integers are there?

A 1 B 2 C 3 D 4 E 5

25. The diagram shows two equilateral triangles. The distance from each point of the smaller triangle to the nearest point of the larger triangle is $\sqrt{3}$, as shown.



What is the difference between the lengths of the edges of the two triangles?

A $2\sqrt{3}$ B $4\frac{1}{2}$ C $3\sqrt{3}$ D 6 E $4\sqrt{3}$

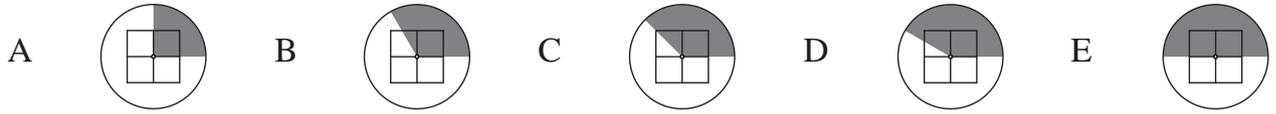


1. What is the value of $\frac{2}{5} + \frac{2}{50} + \frac{2}{500}$?

- A 0.111 B 0.222 C 0.333 D 0.444 E 0.555

2. Each of the diagrams below shows a circle and four small squares. In each case, the centre of the circle is the point where all four squares meet.

In one of the diagrams, exactly one third of the circle is shaded. Which one?



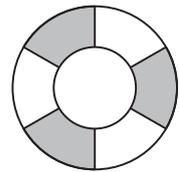
3. How many squares have 7 as their units digit?

- A 0 B 1 C 2 D 3 E 4

4. Which of the following is *not* the sum of two primes?

- A 5 B 7 C 9 D 11 E 13

5. The diagram shows two circles with the same centre. The radius of the outer circle is twice the radius of the inner circle. The region between the inner circle and the outer circle is divided into six equal segments as shown.



What fraction of the area of the outer circle is shaded?

- A $\frac{3}{7}$ B $\frac{3}{8}$ C $\frac{3}{9}$ D $\frac{3}{10}$ E $\frac{3}{11}$

6. The angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6.

What is the difference between the largest angle and the smallest angle?

- A 30° B 40° C 50° D 60° E 70°

7. Four different positive integers are to be chosen so that they have a mean of 2017.

What is the smallest possible range of the chosen integers?

- A 2 B 3 C 4 D 5 E 6

8. Which of the following numbers is the largest?

- A 1.3542 B $1.35\dot{4}2$ C $1.35\dot{4}\dot{2}$ D $1.3\dot{5}4\dot{2}$ E $1.3\dot{5}4\dot{2}$

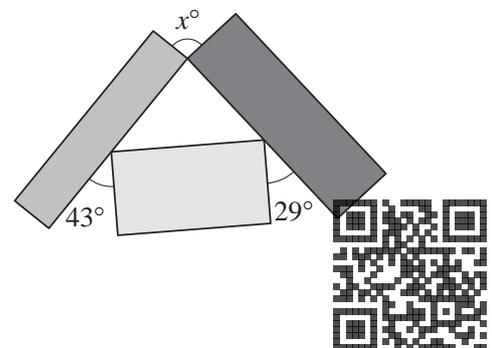
9. The number 'tu' is the two-digit number with units digit u and tens digit t . The digits a and b are distinct, and non-zero. What is the largest possible value of 'ab' - 'ba' ?

- A 81 B 72 C 63 D 54 E 45

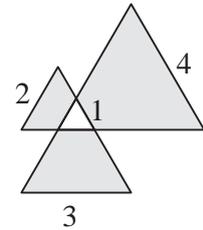
10. The diagram shows three rectangles.

What is the value of x ?

- A 108 B 104 C 100 D 96 E 92



11. The diagram shows four equilateral triangles with sides of lengths 1, 2, 3 and 4. The area of the shaded region is equal to n times the area of the unshaded triangle of side-length 1.



What is the value of n ?

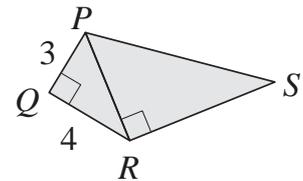
- A 8 B 11 C 18 D 23 E 26

12. The combined age of Alice and Bob is 39. The combined age of Bob and Clare is 40. The combined age of Clare and Dan is 38. The combined age of Dan and Eve is 44. The total of all five ages is 105.

Which of the five is the youngest?

- A Alice B Bob C Clare D Dan E Eve

13. The diagram shows a quadrilateral $PQRS$ made from two similar right-angled triangles, PQR and PSR . The length of PQ is 3, the length of QR is 4 and $\angle PRQ = \angle PSR$.



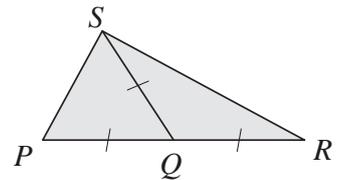
What is the perimeter of $PQRS$?

- A 22 B $22\frac{5}{6}$ C 27 D 32 E $45\frac{1}{3}$

14. For what value of x is 64^x equal to 512^5 ?

- A 6 B 7.5 C 8 D 16 E 40

15. In the diagram shown, $PQ = SQ = QR$ and $\angle SPQ = 2 \times \angle RSQ$. What is the size of angle QRS ?



- A 20° B 25° C 30° D 35° E 40°

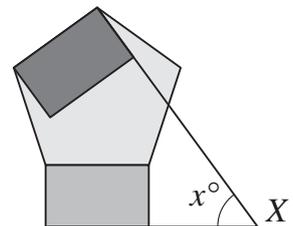
16. The product of two positive integers is equal to twice their sum. This product is also equal to six times the difference between the two integers. What is the sum of these two integers?

- A 6 B 7 C 8 D 9 E 10

17. The diagram shows two rectangles and a regular pentagon. One side of each rectangle has been extended to meet at X .

What is the value of x ?

- A 52 B 54 C 56 D 58 E 60



18. A water tank is $\frac{5}{6}$ full. When 30 litres of water are removed from the tank, the tank is $\frac{4}{5}$ full.

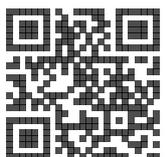
How much water does the tank hold when full?

- A 180 litres B 360 litres C 540 litres D 720 litres E 900 litres

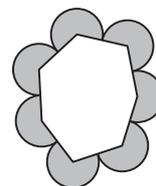
19. $PQRS$ is a square. Point T lies on PQ so that $PT : TQ = 1 : 2$. Point U lies on SR so that $SU : UR = 1 : 2$. The perimeter of $PTUS$ is 40 cm.

What is the area of $PTUS$?

- A 40 cm^2 B 45 cm^2 C 48 cm^2 D 60 cm^2 E 75 cm^2



20. The diagram shows seven circular arcs and a heptagon with equal sides but unequal angles. The sides of the heptagon have length 4. The centre of each arc is a vertex of the heptagon, and the ends of the arc are the midpoints of the two adjacent sides.



What is the total shaded area?

- A 12π B 14π C 16π D 18π E 20π

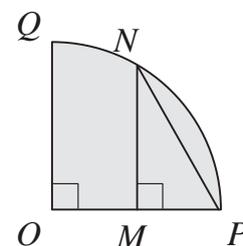
21. *Brachycephalus* frogs are tiny – less than 1 cm long – and have three toes on each foot and two fingers on each ‘hand’, whereas the common frog has five toes on each foot and four fingers on each ‘hand’.

Some *Brachycephalus* and common frogs are in a bucket. Each frog has all its fingers and toes. Between them they have 122 toes and 92 fingers.

How many frogs are in the bucket?

- A 15 B 17 C 19 D 21 E 23

22. The diagram shows an arc PQ of a circle with centre O and radius 8. Angle QOP is a right angle, the point M is the midpoint of OP and N lies on the arc PQ so that MN is perpendicular to OP .



Which of the following is closest to the length of the perimeter of triangle PNM ?

- A 17 B 18 C 19 D 20 E 21

23. Two brothers and three sisters form a single line for a photograph. The two boys refuse to stand next to each other.

How many different line-ups are possible?

- A 24 B 36 C 60 D 72 E 120

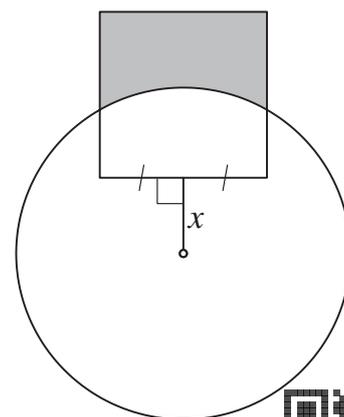
24. The n th term in a certain sequence is calculated by multiplying together all the numbers $\sqrt{1 + \frac{1}{k}}$, where k takes all the integer values from 2 to $n + 1$ inclusive. For example, the third

term in the sequence is $\sqrt{1 + \frac{1}{2}} \times \sqrt{1 + \frac{1}{3}} \times \sqrt{1 + \frac{1}{4}}$.

Which is the smallest value of n for which the n th term of the sequence is an integer?

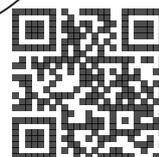
- A 3 B 5 C 6 D 7 E more than 7

25. The diagram shows a circle with radius 2 and a square with sides of length 2. The centre of the circle lies on the perpendicular bisector of a side of the square, at a distance x from the side, as shown. The shaded region – inside the square but outside the circle – has area 2.



What is the value of x ?

- A $\frac{\pi}{3} + \frac{\sqrt{3}}{2} - 1$ B $\frac{\pi}{3} + \frac{\sqrt{3}}{4} - 1$ C $\frac{\pi}{3} + \frac{1}{2}$
 D $\frac{\pi}{3} + 1$ E $\frac{\pi}{3}$



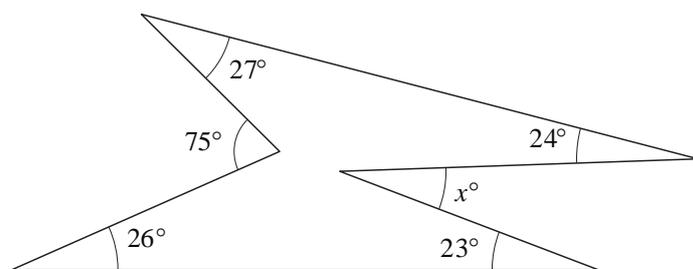
1. What is the value of $6102 - 2016$?
 A 3994 B 4086 C 4096 D 4114 E 4994
2. Which of the following fractions is closest to 1?
 A $\frac{7}{8}$ B $\frac{8}{7}$ C $\frac{9}{10}$ D $\frac{10}{11}$ E $\frac{11}{10}$
3. How many of these five expressions give answers which are *not* prime numbers?
 $1^2 + 2^2$ $2^2 + 3^2$ $3^2 + 4^2$ $4^2 + 5^2$ $5^2 + 6^2$
 A 0 B 1 C 2 D 3 E 4
4. Amrita is baking a cake today. She bakes a cake every fifth day. How many days will it be before she next bakes a cake on a Thursday?
 A 5 B 7 C 14 D 25 E 35
5. When travelling from London to Edinburgh by train, you pass a sign saying 'Edinburgh 200 miles'. Then, $3\frac{1}{2}$ miles later, you pass another sign saying 'Half way between London and Edinburgh'.
 How many miles is it by train from London to Edinburgh?
 A 393 B $396\frac{1}{2}$ C 400 D $403\frac{1}{2}$ E 407

6. One third of the animals in Jacob's flock are goats, the rest are sheep. There are twelve more sheep than goats.

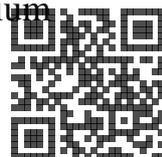
How many animals are there altogether in Jacob's flock?

- A 12 B 24 C 36 D 48 E 60
7. In the diagram, what is the value of x ?

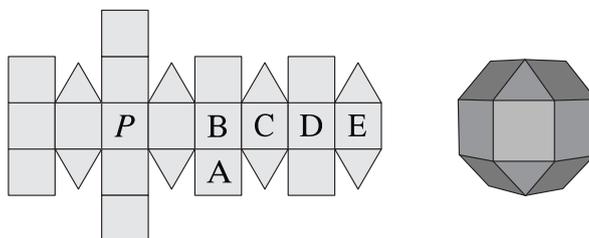
- A 23 B 24 C 25 D 26 E 27



8. What is the value of $2.017 \times 2016 - 10.16 \times 201.7$?
 A 2.016 B 2.017 C 20.16 D 2016 E 2017
9. The world's fastest tortoise is acknowledged to be a leopard tortoise from County Durham called Bertie. In July 2014, Bertie sprinted along a 5.5 m long track in an astonishing 19.6 seconds.
 What was Bertie's approximate average speed in km per hour?
 A 0.1 B 0.5 C 1 D 5 E 10
10. The angles of a quadrilateral taken in order are x° , $5x^\circ$, $2x^\circ$ and $4x^\circ$. Which of the following is the quadrilateral?
 A kite B parallelogram C rhombus D arrowhead E trapezium



11. The net shown consists of squares and equilateral triangles. The net is folded to form a rhombicuboctahedron, as shown.



When the face marked P is placed face down on a table, which face will be facing up?

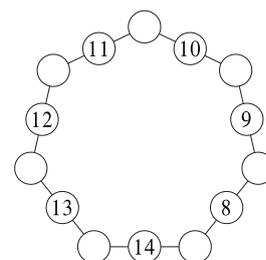
- A B C D E

12. The sum of two numbers a and b is 7 and the difference between them is 2.

What is the value of $a \times b$?

- A $8\frac{1}{4}$ B $9\frac{1}{4}$ C $10\frac{1}{4}$ D $11\frac{1}{4}$ E $12\frac{1}{4}$

13. The diagram shows a heptagon with a line of three circles on each side. Each circle is to contain exactly one number. The numbers 8 to 14 are distributed as shown and the numbers 1 to 7 are to be distributed to the remaining circles. The total of the numbers in each of the lines of three circles is to be the same.



What is this total?

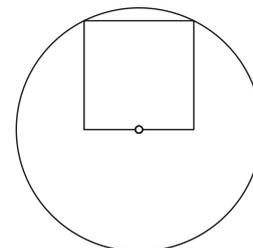
- A 18 B 19 C 20 D 21 E 22

14. Tegwen has the same number of brothers as she has sisters. Each one of her brothers has 50% more sisters than brothers.

How many children are in Tegwen's family?

- A 5 B 7 C 9 D 11 E 13

15. The circle has radius 1 cm. Two vertices of the square lie on the circle. One edge of the square goes through the centre of the circle, as shown.



What is the area of the square?

- A $\frac{4}{5} \text{ cm}^2$ B $\frac{\pi}{5} \text{ cm}^2$ C 1 cm^2 D $\frac{\pi}{4} \text{ cm}^2$ E $\frac{5}{4} \text{ cm}^2$

16. How many of the following positive integers are divisible by 24?

$$2^2 \times 3^2 \times 5^2 \times 7^3$$

$$2^2 \times 3^2 \times 5^3 \times 7^2$$

$$2^2 \times 3^3 \times 5^2 \times 7^2$$

$$2^3 \times 3^2 \times 5^2 \times 7^2$$

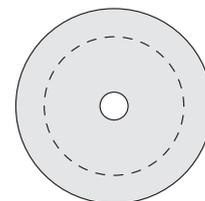
- A 0 B 1 C 2 D 3 E 4

17. The shaded region in the diagram, bounded by two concentric circles, is called an *annulus*. The circles have radii 2 cm and 14 cm.

The dashed circle divides the area of this annulus into two equal areas.

What is its radius?

- A 9 cm B 10 cm C 11 cm D 12 cm E 13 cm



18. The sum of the areas of the squares on the sides of a right-angled isosceles triangle is 72 cm^2 .

What is the area of the triangle?

- A 6 cm^2 B 8 cm^2 C 9 cm^2 D 12 cm^2 E 18 cm^2

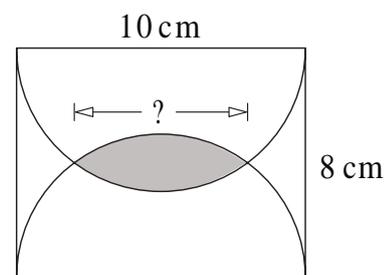
19. A list of positive integers has a median of 8, a mode of 9 and a mean of 10.

What is the smallest possible number of integers in the list?

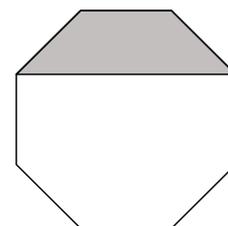
- A 5 B 6 C 7 D 8 E 9



20. Two semicircles are drawn in a rectangle as shown.
What is the width of the overlap of the two semicircles?
A 3 cm B 4 cm C 5 cm D 6 cm E 7 cm



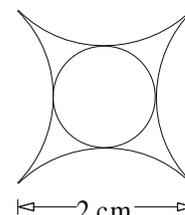
21. The diagram shows a regular octagon. What is the ratio of the area of the shaded trapezium to the area of the whole octagon?
A 1 : 4 B 5 : 16 C 1 : 3 D $\sqrt{2} : 2$ E 3 : 8



22. In a particular group of people, some always tell the truth, the rest always lie. There are 2016 in the group. One day, the group is sitting in a circle. Each person in the group says, "Both the person on my left and the person on my right are liars."
What is the difference between the largest and smallest number of people who could be telling the truth?

A 0 B 72 C 126 D 288 E 336

23. A Saxon silver penny, from the reign of Ethelbert II in the eighth century, was sold in 2014 for £78 000. A design on the coin depicts a circle surrounded by four equal arcs, each a quarter of a circle, as shown. The width of the design is 2 cm.



What is the radius of the small circle, in centimetres?

A $\frac{1}{2}$ B $2 - \sqrt{2}$ C $\frac{1}{2}\sqrt{2}$ D $5 - 3\sqrt{2}$ E $2\sqrt{2} - 2$

24. Every day, Aimee goes up an escalator on her journey to work. If she stands still, it takes her 60 seconds to travel from the bottom to the top. One day the escalator was broken so she had to walk up it. This took her 90 seconds.

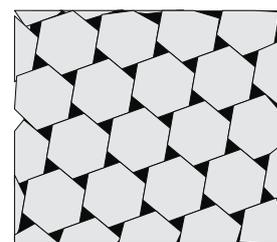
How many seconds would it take her to travel up the escalator if she walked up at the same speed as before while it was working?

A 30 B 32 C 36 D 45 E 75

25. The tiling pattern shown uses two types of tile, regular hexagons and equilateral triangles, with the length of each side of the equilateral triangles equal to half the length of each side of the hexagons. A large number of tiles is used to cover a floor.

Which of the following is closest to the fraction of the floor that is shaded black?

A $\frac{1}{8}$ B $\frac{1}{10}$ C $\frac{1}{12}$ D $\frac{1}{13}$ E $\frac{1}{16}$



Part III

Senior Challenge Past Paper Collection

Answers

Last updated: April 20, 2020

	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	
1	A	D	D	C	E	C	C	A	C	D	E	A	C	D	B	C	C	D			1
2	B	A	E	B	A	A	B	D	B	D	B	C	B	A	D	B	C	B			2
3	C	A	D	D	A	B	D	E	C	B	D	D	D	B	B	E	D	A			3
4	C	C	D	A	C	C	C	E	B	D	B	C	A	B	C	E	E	B			4
5	E	A	B	E	A	E	E	A	D	B	C	E	C	A	A	C	B	B			5
6	C	B	E	B	D	A	E	C	B	C	C	E	B	E	D	B	B	D			6
7	B	C	C	E	B	D	D	B	C	D	D	B	B	B	A	C	A	D			7
8	D	E	C	D	C	E	A	C	D	C	C	B	C	D	B	B	B	B			8
9	A	B	D	C	C	E	D	D	D	B	C	A	D	B	C	B	A	D			9
10	E	D	B	B	B	B	D	B	E	C	E	B	C	D	B	D	D	C			10
11	A	C	A	A	E	D	B	C	D	C	D	A	E	C	E	D	B	C			11
12	A	C	C	D	E	E	C	E	A	D	E	B	D	A	C	B	A	E			12
13	D	A	A	D	D	B	C	C	A	D	B	D	E	E	A	C	E	D			13
14	E	E	C	C	B	B	D	D	E	C	D	D	E	E	D	A	D	C			14
15	D	D	C	E	C	D	A	D	E	B	A	B	C	C	B	B	D	B			15
16	E	A	E	B	C	A	A	B	E	E	A	D	A	A	D	D	D	A			16
17	D	C	A	E	E	C	E	C	E	A	B	A	C	C	D	D	B	E			17
18	B	E	A	A	D	A	C	A	A	E	A	C	D	D	E	E	A	E			18
19	C	E	B	A	C	D	B	C	B	B	E	C	E	A	D	B	C	A			19
20	A	B	E	C	D	C	B	E	C	B	E	E	A	E	E	B	E	A			20
21	D	B	B	E	B	B	B	B	A	C	D	B	B	C	C	C	A	D			21
22	B	C	B	C	A	D	A	C	B	A	B	C	A	B	C	D	C	C			22
23	E	B	D	B	D	E	B	D	B	B	C	A	E	D	E	C	B	B			23
24	B	A	E	D	A	D	E	B	E	B	B	E	A	C	B	D	E	E			24
25	C	D	C	A	E	D	D	A	D	C	B	D	B	C	D	A	C	A			25



1. What is the value of $123^2 - 23^2$?

- A 10 000 B 10 409 C 12 323 D 14 600 E 15 658

2. What is the value of $(2019 - (2000 - (10 - 9))) - (2000 - (10 - (9 - 2019)))$?

- A 4040 B 40 C -400 D -4002 E -4020

3. Used in measuring the width of a wire, one mil is equal to one thousandth of an inch. An inch is about 2.5 cm.

Which of these is approximately equal to one mil?

- A $\frac{1}{40}$ mm B $\frac{1}{25}$ mm C $\frac{1}{4}$ mm D 25 mm E 40 mm

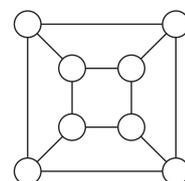
4. For how many positive integer values of n is $n^2 + 2n$ prime?

- A 0 B 1 C 2 D 3 E more than 3

5. Olive Green wishes to colour all the circles in the diagram so that, for each circle, there is exactly one circle of the same colour joined to it.

What is the smallest number of colours that Olive needs to complete this task?

- A 1 B 2 C 3 D 4 E 5



6. Each of the factors of 100 is to be placed in a 3 by 3 grid, one per cell, in such a way that the products of the three numbers in each row, column and diagonal are all equal. The positions of the numbers 1, 2, 50 and x are shown in the diagram.

What is the value of x ?

- A 4 B 5 C 10 D 20 E 25

x	1	50
2		

7. Lucy is asked to choose p , q , r and s to be the numbers 1, 2, 3 and 4, in some order, so as to make the value of $\frac{p}{q} + \frac{r}{s}$ as small as possible.

What is the smallest value Lucy can achieve in this way?

- A $\frac{7}{12}$ B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{5}{6}$ E $\frac{11}{12}$

8. The number x is the solution to the equation $3^{(3^x)} = 333$.

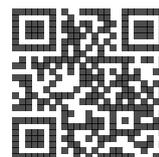
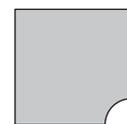
Which of the following is true?

- A $0 < x < 1$ B $1 < x < 2$ C $2 < x < 3$ D $3 < x < 4$ E $4 < x < 5$

9. A square of paper is folded in half four times to obtain a smaller square. Then a corner is removed as shown.

Which of the following could be the paper after it is unfolded?

- A  B  C  D  E 



10. Which of the following five values of n is a counterexample to the statement in the box below?

For a positive integer n , at least one of $6n - 1$ and $6n + 1$ is prime.

A 10 B 19 C 20 D 21 E 30

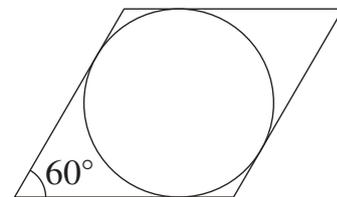
11. For how many integer values of k is $\sqrt{200 - \sqrt{k}}$ also an integer?

A 11 B 13 C 15 D 17 E 20

12. A circle with radius 1 touches the sides of a rhombus, as shown. Each of the smaller angles between the sides of the rhombus is 60° .

What is the area of the rhombus?

A 6 B 4 C $2\sqrt{3}$ D $3\sqrt{3}$ E $\frac{8\sqrt{3}}{3}$



13. Anish has a number of small congruent square tiles to use in a mosaic. When he forms the tiles into a square of side n , he has 64 tiles left over. When he tries to form the tiles into a square of side $n + 1$, he has 25 too few.

How many tiles does Anish have?

A 89 B 1935 C 1980 D 2000 E 2019

14. One of the following is the largest square that is a factor of $10!$. Which one?

Note that, $n! = 1 \times 2 \times 3 \times \dots \times (n - 1) \times n$.

A $(4!)^2$ B $(5!)^2$ C $(6!)^2$ D $(7!)^2$ E $(8!)^2$

15. The highest common factors of all the pairs chosen from the positive integers Q , R and S are three different primes.

What is the smallest possible value of $Q + R + S$?

A 41 B 31 C 30 D 21 E 10

16. The numbers x , y and z satisfy the equations $9x + 3y - 5z = -4$ and $5x + 2y - 2z = 13$.

What is the mean of x , y and z ?

A 10 B 11 C 12 D 13 E 14

17. Jeroen writes a list of 2019 consecutive integers. The sum of his integers is 2019.

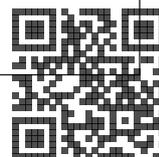
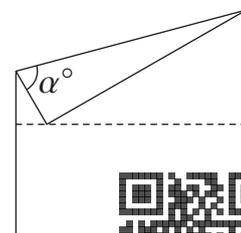
What is the product of all the integers in Jeroen's list?

A 2019^2 B $\frac{2019 \times 2020}{2}$ C 2^{2019} D 2019 E 0

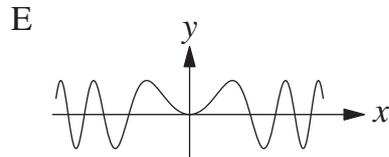
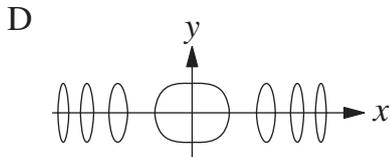
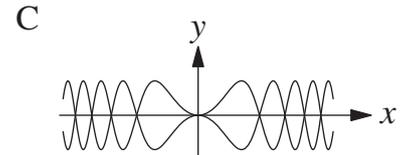
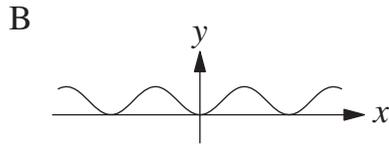
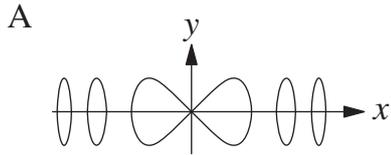
18. Alison folds a square piece of paper in half along the dashed line shown in the diagram. After opening the paper out again, she then folds one of the corners onto the dashed line.

What is the value of α ?

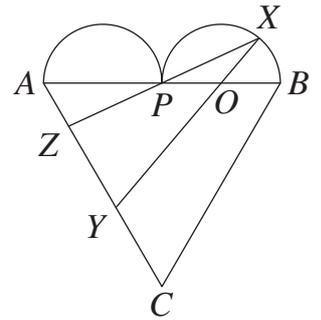
A 45 B 60 C 65 D 70 E 75



19. Which of the following could be the graph of $y^2 = \sin(x^2)$?



20. The "heart" shown in the diagram is formed from an equilateral triangle ABC and two congruent semicircles on AB . The two semicircles meet at the point P . The point O is the centre of one of the semicircles. On the semicircle with centre O , lies a point X . The lines XO and XP are extended to meet AC at Y and Z respectively. The lines XY and XZ are of equal length.



What is $\angle ZXY$?

- A 20° B 25° C 30° D 40° E 45°

21. In a square garden $PQRT$ of side 10 m, a ladybird sets off from Q and moves along edge QR at 30 cm per minute. At the same time, a spider sets off from R and moves along edge RT at 40 cm per minute. What will be the shortest distance between them, in metres?

- A 5 B 6 C $5\sqrt{2}$ D 8 E 10

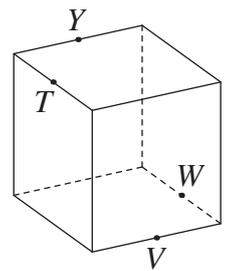
22. A function f satisfies the equation $(n - 2019)f(n) - f(2019 - n) = 2019$ for every integer n . What is the value of $f(2019)$?

- A 0 B 1 C 2018×2019 D 2019^2 E 2019×2020

23. The edge-length of the solid cube shown is 2. A single plane cut goes through the points Y, T, V and W which are midpoints of the edges of the cube, as shown.

What is the area of the cross-section?

- A $\sqrt{3}$ B $3\sqrt{3}$ C 6 D $6\sqrt{2}$ E 8



24. The numbers x, y and z are given by $x = \sqrt{12 - 3\sqrt{7}} - \sqrt{12 + 3\sqrt{7}}$, $y = \sqrt{7 - 4\sqrt{3}} - \sqrt{7 + 4\sqrt{3}}$ and $z = \sqrt{2 + \sqrt{3}} - \sqrt{2 - \sqrt{3}}$.

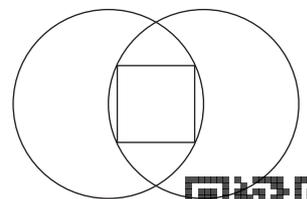
What is the value of xyz ?

- A 1 B -6 C -8 D 18 E 12

25. Two circles of radius 1 are such that the centre of each circle lies on the other circle. A square is inscribed in the space between the circles.

What is the area of the square?

- A $2 - \frac{\sqrt{7}}{2}$ B $2 + \frac{\sqrt{7}}{2}$ C $4 - \sqrt{5}$ D 1 E $\frac{\sqrt{5}}{5}$



1. When the following are evaluated, how many of the answers are odd numbers?

$$1^2, 2^3, 3^4, 4^5, 5^6$$

A 1 B 2 C 3 D 4 E 5

2. The positive integer 2018 is the product of two primes.

What is the sum of these two primes?

A 1001 B 1010 C 1011 D 1100 E 1101

3. Which of the following shows the digit 6 after it has been rotated clockwise through 135° ?

A  B  C  D  E 

4. Which of the following is not a multiple of 5?

A $2019^2 - 2014^2$ B $2019^2 \times 10^2$ C $2020^2 \div 101^2$ D $2010^2 - 2005^2$
E $2015^2 \div 5^2$

5. Which of the following numbers is the largest?

A $\frac{397}{101}$ B $\frac{487}{121}$ C $\frac{596}{153}$ D $\frac{678}{173}$ E $\frac{796}{203}$

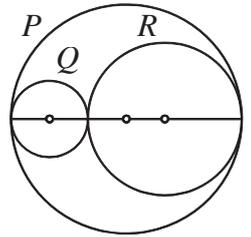
6. Which of the following is equal to $25 \times 15 \times 9 \times 5.4 \times 3.24$?

A 3^9 B 3^{10} C 3^{11} D 3^{14} E 3^{17}

7. The circles P , Q and R are all tangent to each other. Their centres all lie on a diameter of P , as shown in the figure.

What is the value of $\frac{\text{circumference of } Q + \text{circumference of } R}{\text{circumference of } P}$?

A 1 B $\frac{1}{2}$ C $\frac{1}{3}$ D $\frac{1}{4}$
E more information needed



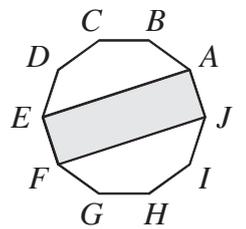
8. What are the last two digits of 7^{2018} ?

A 07 B 49 C 43 D 01 E 18

9. The diagram shows a rectangle $AEFJ$ inside a regular decagon $ABCDEFGHIJ$.

What is the ratio of the area of the rectangle to the area of the decagon?

A 2 : 5 B 1 : 4 C 3 : 5 D 3 : 10 E 3 : 20



10. On a training ride, Laura averages speeds of 12 km/h for 5 minutes, then 15 km/h for 10 minutes and finally 18 km/h for 15 minutes.

What was her average speed over the whole ride?

A 13 km/h B 14 km/h C 15 km/h D 16 km/h E 17 km/h

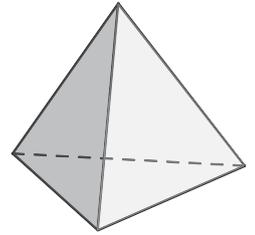
11. How many of the following four equations has a graph that does *not* pass through the origin?

$$y = x^4 + 1 \quad y = x^4 + x \quad y = x^4 + x^2 \quad y = x^4 + x^3$$

A 0 B 1 C 2 D 3 E 4



12. A regular tetrahedron is a polyhedron with four faces, each of which is an equilateral triangle, as shown. A solid regular tetrahedron is cut into two pieces by a single plane cut.



Which of the following could *not* be the shape of the section formed by the cut?

- A a pentagon
 B a square
 C a rectangle that is not a square
 D a trapezium
 E a triangle that is not equilateral
13. The lines $y = x$ and $y = mx - 4$ intersect at the point P .

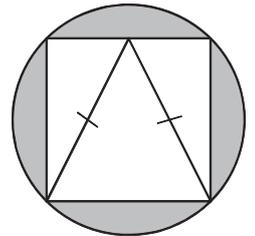
What is the sum of the positive integer values of m for which the coordinates of P are also positive integers?

- A 3
 B 5
 C 7
 D 8
 E 10
14. The following twelve integers are written in ascending order:

1, x , x , x , y , y , y , y , y , 8, 9, 11.

The mean of these twelve integers is 7. What is the median?

- A 6
 B 7
 C 7.5
 D 8
 E 9
15. A square is inscribed in a circle of radius 1. An isosceles triangle is inscribed in the square as shown.



What is the ratio of the area of this triangle to the area of the shaded region?

- A $\pi : \sqrt{2}$
 B $\pi : 1$
 C $1 : 4$
 D $1 : \pi - 2$
 E $2 : \pi$
16. The numbers p , q , r and s satisfy the following equations:

$$p + 2q + 3r + 4s = k \quad 4p = 3q = 2r = s.$$

What is the smallest value of k for which p , q , r and s are all positive integers?

- A 20
 B 24
 C 25
 D 77
 E 154
17. Bethany has 11 pound coins and some 20p coins and some 50p coins in her purse. The mean value of the coins is 52 pence.

Which could not be the number of coins in the purse?

- A 35
 B 40
 C 50
 D 65
 E 95
18. P , Q and R are the three angles of a triangle, when each has been rounded to the nearest degree.

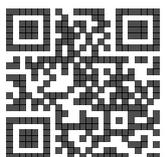
Which of the following is the complete list of possible values of $P + Q + R$?

- A 179° , 180° or 181°
 B 180° , 181° or 182°
 C 178° , 179° or 180°
 D 180°
 E 178° , 179° , 180° , 181° or 182°

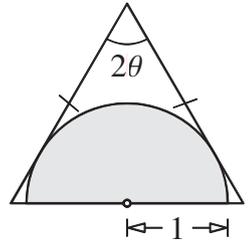
19. How many pairs of numbers (m, n) are there such that the following statement is true?

‘A regular m -sided polygon has an exterior angle of size n° and a regular n -sided polygon has an exterior angle of size m° .’

- A 24
 B 22
 C 20
 D 18
 E 16



20. The diagram shows a semicircle of radius 1 inside an isosceles triangle. The diameter of the semicircle lies along the 'base' of the triangle, and the angle of the triangle opposite the 'base' is equal to 2θ . Each of the two equal sides of the triangle is tangent to the semicircle.



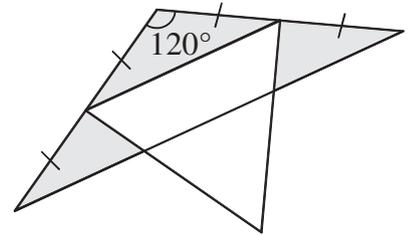
What is the area of the triangle?

- A $\frac{1}{2} \tan 2\theta$ B $\sin \theta \cos \theta$ C $\sin \theta + \cos \theta$ D $\frac{1}{2} \cos 2\theta$
 E $\frac{1}{\sin \theta \cos \theta}$
21. The graph of $y = \frac{1}{x}$ is reflected in the line $y = 1$. The resulting image is reflected in the line $y = -x$.

What is the equation of the final graph?

- A $y = \frac{-1}{(x+2)}$ B $y = \frac{1}{(x-1)}$ C $y = \frac{1}{(x-2)}$ D $y = \frac{-1}{(x-1)}$ E $y = \frac{-1}{(x-2)}$

22. The diagram shows two overlapping triangles; an isosceles triangle with an angle of 120° and an equilateral triangle with area 36. Two of the vertices of the equilateral triangle are midpoints of the equal sides of the isosceles triangle.

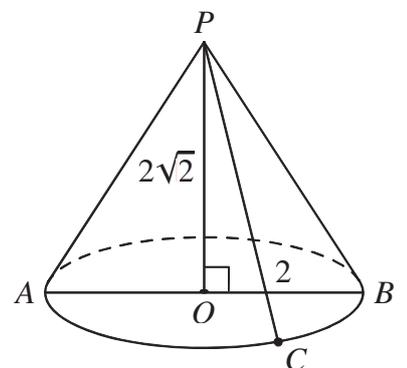


What is the total area of the shaded regions (inside the isosceles triangle but outside the equilateral triangle)?

- A 24 B 26 C 28 D 30 E 32
23. For particular real numbers a and b , the function f is defined by $f(x) = ax + b$, and is such that $f(f(f(x))) = 27x - 52$.

Which of the following formulas defines the function g such that, for all values of x , $g(f(x)) = x$?

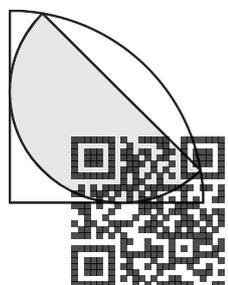
- A $\frac{1}{3}x - 4$ B $\frac{1}{3}x + \frac{4}{3}$ C $4x - 3$ D $\frac{1}{3}x - \frac{4}{3}$ E $3x - 4$
24. The diagram shows a circle with centre O which lies in a horizontal plane. The diameter AB has length 4. Point P lies vertically above O and $PO = 2\sqrt{2}$. Point C lies on the semicircular arc AB such that the ratio of the lengths of the arcs AC and CB is 2 : 1.



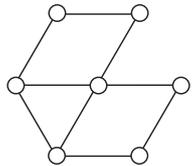
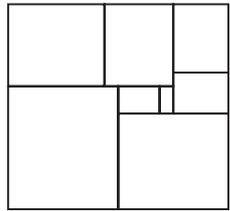
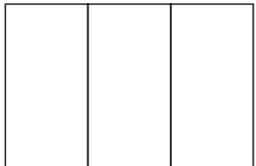
What is the shortest distance from A to PC ?

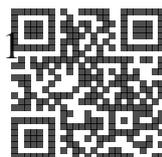
- A $\sqrt{2}$ B $\sqrt{3}$ C 2 D $2\sqrt{2}$ E 3
25. A semicircle is inscribed in a quarter circle as shown.

What fraction of the quarter circle is shaded?



- A $\frac{1}{3}$ B $\frac{1}{\sqrt{3}}$ C $\frac{2}{3}$ D $\frac{\sqrt{3}}{2}$ E $\frac{1}{\sqrt{2}}$

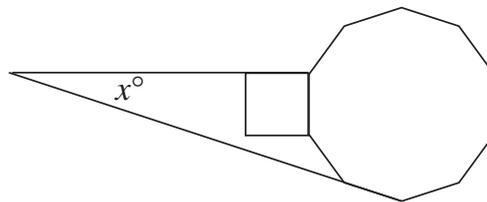
1. One of the following numbers is prime. Which is it?
 A $2017 - 2$ B $2017 - 1$ C 2017 D $2017 + 1$ E $2017 + 2$
2. Last year, an earthworm from Wigan named Dave wriggled into the record books as the largest found in the UK. Dave was 40 cm long and had a mass of 26 g. What was Dave's mass per unit length?
 A 0.6 g/cm B 0.65 g/cm C 0.75 g/cm D 1.6 g/cm E 1.75 g/cm
3. The five integers 2, 5, 6, 9, 14 are arranged into a different order. In the new arrangement, the sum of the first three integers is equal to the sum of the last three integers. What is the middle number in the new arrangement?
 A 2 B 5 C 6 D 9 E 14
4. Which of the following is equal to $2017 - \frac{1}{2017}$?
 A $\frac{2017^2}{2016}$ B $\frac{2016}{2017}$ C $\frac{2018}{2017}$ D $\frac{4059}{2017}$ E $\frac{2018 \times 2016}{2017}$
5. One light-year is nearly 6×10^{12} miles. In 2016, the Hubble Space Telescope set a new cosmic record, observing a galaxy 13.4 thousand million light-years away. Roughly how many miles is that?
 A 8×10^{20} B 8×10^{21} C 8×10^{22} D 8×10^{23} E 8×10^{24}
6. The circles in the diagram are to be coloured so that any two circles connected by a line segment have different colours. What is the smallest number of colours required?
 A 2 B 3 C 4 D 5 E 6
- 
7. The positive integer k satisfies the equation $\sqrt{2} + \sqrt{8} + \sqrt{18} = \sqrt{k}$. What is the value of k ?
 A 28 B 36 C 72 D 128 E 288
8. When evaluated, which of the following is not an integer?
 A 1^{-1} B $4^{-\frac{1}{2}}$ C 6^0 D $8^{\frac{2}{3}}$ E $16^{\frac{3}{4}}$
9. The diagram shows an $n \times (n + 1)$ rectangle tiled with $k \times (k + 1)$ rectangles, where n and k are integers and k takes each value from 1 to 8 inclusive. What is the value of n ?
 A 16 B 15 C 14 D 13 E 12
- 
10. A rectangle is divided into three smaller congruent rectangles as shown. Each smaller rectangle is similar to the large rectangle. In each of these four rectangles, what is the ratio of the length of a longer side to that of a shorter side?
 A $2\sqrt{3} : 1$ B $3 : 1$ C $2 : 1$ D $\sqrt{3} : 1$ E $\sqrt{2} : 1$
- 



11. The teenagers Sam and Jo notice the following facts about their ages:
The difference between the squares of their ages is four times the sum of their ages.
The sum of their ages is eight times the difference between their ages.
What is the age of the older of the two?

A 15 B 16 C 17 D 18 E 19

12. The diagram shows a square and a regular decagon that share an edge. One side of the square is extended to meet an extended side of the decagon.



What is the value of x ?

A 15 B 18 C 21 D 24 E 27

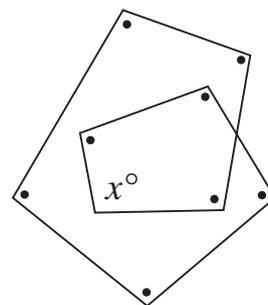
13. Isobel: "Josh is innocent" Genotan: "Tegan is guilty"
Josh: "Genotan is guilty" Tegan: "Isobel is innocent"
Only the guilty person is lying; all the others are telling the truth. Who is guilty?

A Isobel B Josh C Genotan D Tegan E More information required

14. In the diagram, all the angles marked \bullet are equal in size to the angle marked x° .

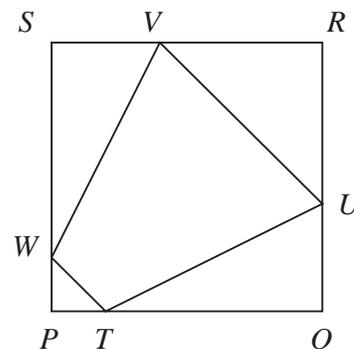
What is the value of x ?

A 100 B 105 C 110 D 115 E 120



15. The diagram shows a square $PQRS$. Points T , U , V and W lie on the edges of the square as shown, such that $PT = 1$, $QU = 2$, $RV = 3$ and $SW = 4$. The area of $TUVW$ is half that of $PQRS$. What is the length of PQ ?

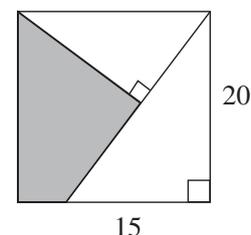
A 5 B 6 C 7 D 8 E 9



16. The diagram shows two right-angled triangles inside a square. The perpendicular edges of the larger triangle have lengths 15 and 20.

What is the area of the shaded quadrilateral?

A 142 B 146 C 150 D 154 E 158



17. Amy, Beth and Claire each has some sweets. Amy gives one third of her sweets to Beth. Beth gives one third of all the sweets she now has to Claire. Then Claire gives one third of all the sweets she now has to Amy. All the girls end up having the same number of sweets.

Claire begins with 40 sweets. How many sweets does Beth have originally?

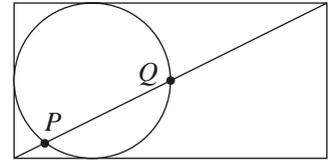
A 20 B 30 C 40 D 50 E 60



18. The arithmetic mean, A , of any two positive numbers x and y is defined to be $A = \frac{1}{2}(x + y)$ and their geometric mean, G , is defined to be $G = \sqrt{xy}$. For two particular values x and y , with $x > y$, the ratio $A : G = 5 : 4$. For these values of x and y , what is the ratio $x : y$?

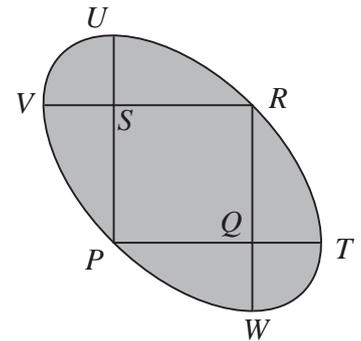
A 5 : 4 B 2 : 1 C 5 : 2 D 7 : 2 E 4 : 1

19. The diagram shows a circle of radius 1 touching three sides of a 2×4 rectangle. A diagonal of the rectangle intersects the circle at P and Q , as shown. What is the length of the chord PQ ?



A $\sqrt{5}$ B $\frac{4}{\sqrt{5}}$ C $\sqrt{5} - \frac{2}{\sqrt{5}}$ D $\frac{5\sqrt{5}}{6}$ E 2

20. The diagram shows a square $PQRS$ with edges of length 1, and four arcs, each of which is a quarter of a circle. Arc TRU has centre P ; arc VPW has centre R ; arc UV has centre S ; and arc WT has centre Q .



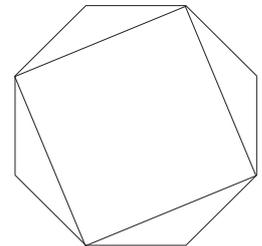
What is the length of the perimeter of the shaded region?

A 6 B $(2\sqrt{2} - 1)\pi$ C $(\sqrt{2} - \frac{1}{2})\pi$
D 2π E $(3\sqrt{2} - 2)\pi$

21. How many pairs (x, y) of positive integers satisfy the equation $4^x = y^2 + 15$?

A 0 B 1 C 2 D 4 E an infinite number

22. The diagram shows a regular octagon and a square formed by drawing four diagonals of the octagon. The edges of the square have length 1.



What is the area of the octagon?

A $\frac{\sqrt{6}}{2}$ B $\frac{4}{3}$ C $\frac{7}{5}$ D $\sqrt{2}$ E $\frac{3}{2}$

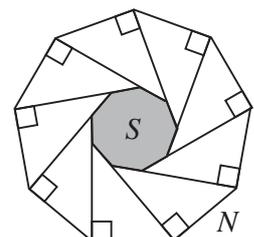
23. The parabola with equation $y = x^2$ is reflected in the line with equation $y = x + 2$. Which of the following is the equation of the reflected parabola?

A $x = y^2 + 4y + 2$ B $x = y^2 + 4y - 2$ C $x = y^2 - 4y + 2$
D $x = y^2 - 4y - 2$ E $x = y^2 + 2$

24. There is a set of straight lines in a plane such that each line intersects exactly ten others. Which of the following could not be the number of lines in that set?

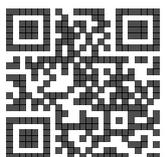
A 11 B 12 C 15 D 16 E 20

25. The diagram shows a regular nonagon N . Moving clockwise around N , at each vertex a line segment is drawn perpendicular to the preceding edge. This produces a smaller nonagon S , shown shaded.



What fraction of the area of N is the area of S ?

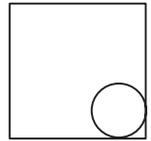
A $\frac{1 - \cos 40^\circ}{1 + \cos 40^\circ}$ B $\frac{\cos 40^\circ}{1 + \cos 40^\circ}$ C $\frac{\sin 40^\circ}{1 + \sin 40^\circ}$ D $\frac{1 - \sin 40^\circ}{1 + \sin 40^\circ}$ E $\frac{1}{9}$



1. How many times does the digit 9 appear in the answer to 987654321×9 ?
 A 0 B 1 C 5 D 8 E 9
2. On a Monday, all prices in Isla's shop are 10% more than normal. On Friday all prices in Isla's shop are 10% less than normal. James bought a book on Monday for £5.50. What would be the price of another copy of this book on Friday?

A £5.50 B £5.00 C £4.95 D £4.50 E £4.40

3. The diagram shows a circle with radius 1 that rolls without slipping around the inside of a square with sides of length 5. The circle rolls once around the square, returning to its starting point. What distance does the centre of the circle travel?

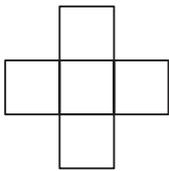


A $16 - 2\pi$ B 12 C $6 + \pi$ D $20 - 2\pi$ E 20

4. Alex draws a scalene triangle. One of the angles is 80° . Which of the following could be the difference between the other two angles in Alex's triangle?

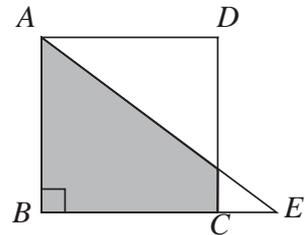
A 0° B 60° C 80° D 100° E 120°

5. All the digits 2, 3, 4, 5 and 6 are placed in the grid, one in each cell, to form two three-digit numbers that are squares. Which digit is placed in the centre of the grid?



A 2 B 3 C 4 D 5 E 6

6. The diagram shows a square $ABCD$ and a right-angled triangle ABE . The length of BC is 3. The length of BE is 4.



What is the area of the shaded region?

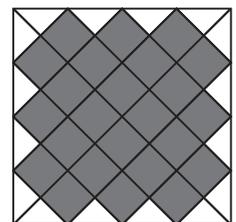
A $5\frac{1}{4}$ B $5\frac{3}{8}$ C $5\frac{1}{2}$ D $5\frac{5}{8}$ E $5\frac{3}{4}$

7. Which of these has the smallest value?

A 2016^{-1} B $2016^{-1/2}$ C 2016^0 D $2016^{1/2}$ E 2016^1

8. Points are drawn on the sides of a square, dividing each side into n equal parts (so, in the example shown, $n = 4$).

The points are joined in the manner indicated, to form several small squares (24 in the example, shown shaded) and some triangles.



How many small squares are formed when $n = 7$?

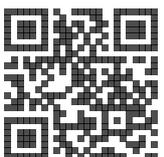
A 56 B 84 C 140 D 840 E 5040

9. A square has vertices at $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$. Graphs of the following equations are drawn on the same set of axes as the square.

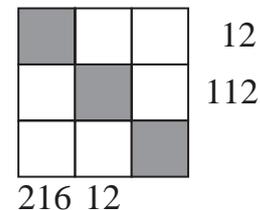
$$x^2 + y^2 = 1, \quad y = x + 1, \quad y = -x^2 + 1, \quad y = x, \quad y = \frac{1}{x}$$

How many of the graphs pass through exactly two of the vertices of the square?

A 1 B 2 C 3 D 4 E 5



10. The digits from 1 to 9 are to be written in the nine cells of the 3×3 grid shown, one digit in each cell.



The product of the three digits in the first row is 12.

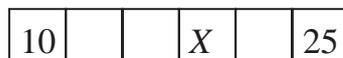
The product of the three digits in the second row is 112.

The product of the three digits in the first column is 216.

The product of the three digits in the second column is 12.

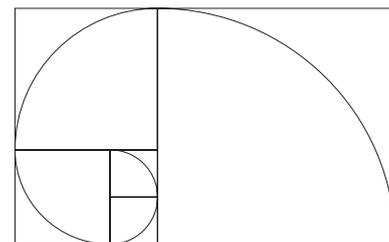
What is the product of the digits in the shaded cells?

- A 24 B 30 C 36 D 48 E 140
11. In the grid below each of the blank squares and the square marked X are to be filled by the mean of the two numbers in its adjacent squares. Which number should go in the square marked X ?



- A 15 B 16 C 17 D 18 E 19
12. What is the smallest square that has 2016 as a factor?
- A 42^2 B 84^2 C 168^2 D 336^2 E 2016^2

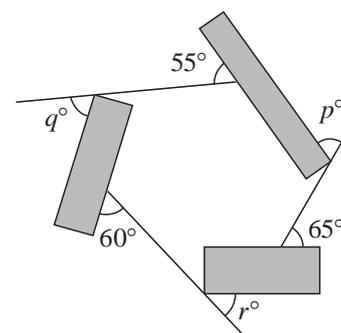
13. Five square tiles are put together side by side. A quarter circle is drawn on each tile to make a continuous curve as shown. Each of the smallest squares has side-length 1. What is the total length of the curve?



- A 6π B 6.5π C 7π D 7.5π E 8π
14. Which of the following values of the positive integer n is a counterexample to the statement: "If n is not prime then $n - 2$ is not prime" ?
- A 6 B 11 C 27 D 33 E 51

15. The diagram shows three rectangles and three straight lines.

What is the value of $p + q + r$?

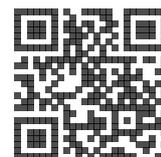


- A 135 B 180 C 210
D 225 E 270

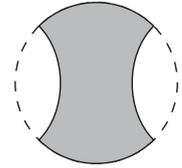
16. For which value of k is $\sqrt{2016} + \sqrt{56}$ equal to 14^k ?

- A $\frac{1}{2}$ B $\frac{3}{4}$ C $\frac{5}{4}$ D $\frac{3}{2}$ E $\frac{5}{2}$
17. Aaron has to choose a three-digit code for his bike lock. The digits can be chosen from 1 to 9. To help him remember them, Aaron chooses three different digits in increasing order, for example 278. How many such codes can be chosen?

- A 779 B 504 C 168 D 84 E 9



18. The circumference of a circle with radius 1 is divided into four equal arcs. Two of the arcs are 'turned over' as shown. What is the area of the shaded region?

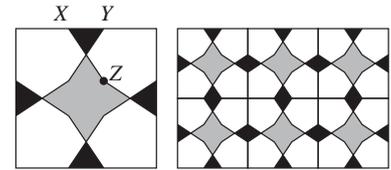


- A 1 B $\sqrt{2}$ C $\frac{1}{2}\pi$ D $\sqrt{3}$ E 2
19. Let S be a set of five different positive integers, the largest of which is m . It is impossible to construct a quadrilateral with non-zero area, whose side-lengths are all distinct elements of S . What is the smallest possible value of m ?

- A 2 B 4 C 9 D 11 E 12

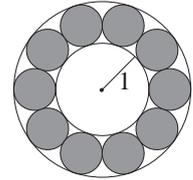
20. Michael was walking in Marrakesh when he saw a tiling formed by tessellating the square tile as shown.

The tile has four lines of symmetry and the length of each side is 8 cm. The length of XY is 2 cm. The point Z is such that XZ is a straight line and YZ is parallel to sides of the square.



What is the area of the central grey octagon?

- A 6 cm^2 B 7 cm^2 C 8 cm^2 D 9 cm^2 E 10 cm^2
21. The diagram shows ten equal discs that lie between two concentric circles – an inner circle and an outer circle. Each disc touches two neighbouring discs and both circles. The inner circle has radius 1. What is the radius of the *outer* circle?



- A $2 \tan 36^\circ$ B $\frac{\sin 36^\circ}{1 - \sin 36^\circ}$ C $\frac{1 + \sin 18^\circ}{1 - \sin 18^\circ}$ D $\frac{2}{\cos 18^\circ}$ E $\frac{9}{5}$

22. Three friends make the following statements.

Ben says, "Exactly one of Dan and Cam is telling the truth."

Dan says, "Exactly one of Ben and Cam is telling the truth."

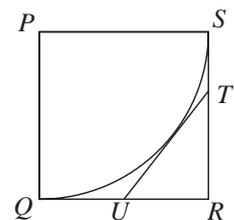
Cam says, "Neither Ben nor Dan is telling the truth."

Which of the three friends is lying?

- A Just Ben B Just Dan C Just Cam D Each of Ben and Cam
E Each of Ben, Cam and Dan
23. A cuboid has sides of lengths 22, 2 and 10. It is contained within a sphere of the smallest possible radius. What is the side-length of the largest cube that will fit inside the same sphere?

- A 10 B 11 C 12 D 13 E 14

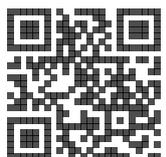
24. The diagram shows a square $PQRS$. The arc QS is a quarter circle. The point U is the midpoint of QR and the point T lies on SR . The line TU is a tangent to the arc QS . What is the ratio of the length of TR to the length of UR ?



- A 3 : 2 B 4 : 3 C 5 : 4 D 7 : 6 E 9 : 8

25. Let n be the smallest integer for which $7n$ has 2016 digits. What is the units digit of n ?

- A 0 B 1 C 4 D 6 E 8



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